Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation. Keep your work neat and tidy, use complete sentences and correct punctuation, and please staple your assignment. Do not use pages torn from a coil notebook, or if you do, please trim off the ragged edge.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort. Instead, discuss solutions among your peers or see me for hints.

Late Assignment Policy: Assignments handed in t days late will incur a penalty of 4^t %. Here t is rounded up to the nearest integer.

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1. Let S be an ordered set, $A \subset S$ and suppose b is an upper bound for A. If $b \in A$ show that $b = \sup(A)$.

2. Let S be an ordered set and $A \subset S$ be a nonempty subset that is bounded above. Suppose $\sup(A)$ exists and $sup(A) \notin A$. Show that A contains a countably infinite subset.

3. Suppose that $S \subset \mathbb{R}$ and that S is nonempty and bounded. Show that for every $\epsilon > 0$ there is some $x \in S$ such that
$$\sup(S) - \epsilon < x \leq \sup(S)$$
(The proof is very short!)

4. Let F be an ordered field and $x, y \in F$. If $0 < x < y$ show that $x^2 < y^2$. (Use only the properties of ordered fields here: Definition 1.1.7 and Proposition 1.1.8 of the textbook.)

5. Let F be an ordered field and $x, y \in F$. Show that $x^2 + y^2 = 0$ if and only if $x = 0$ and $y = 0$. Again, use only properties of ordered fields here.

6. Let $x \geq 0$ be a real number. Show that there exists $n \in \mathbb{N}$ such that $n - 1 \leq x < n$.

7. Let $A \subset \mathbb{R}$ and $B \subset \mathbb{R}$ be bounded and nonempty. Let $C = \{a + b \mid a \in A, b \in B\}$. Show that $\sup(C) = \sup(A) + \sup(B)$.

8. Prove that for real numbers x and y, $\max\{x, y\} = \dfrac{x + y + |x - y|}{2}$.