Note: Structure your proofs by stating the full "**Proposition:**" followed by "**Proof:**" as in class, and please start each new proof on a new page. Indent your argument where appropriate for readability and be careful with notation. Keep your work neat and tidy, use complete sentences and correct punctuation, and please staple your assignment. Do not use pages torn from a coil notebook, or if you do, please trim off the ragged edge.

As with any proof, clarity of presentation is as important as solving the problem. Strive to make your proofs clear, concise and precise. Feel free to use results from earlier problems in subsequent ones.

Finally, proofs for many of the problems in this course are likely available online if one looks hard enough, but the point of this course is to learn analysis, and in particular how to write analysis proofs. As such, please resist turning to the internet for help except as a last resort; instead, discuss solutions among your peers or see me for hints.

Late Assignment Policy: Assignments handed in t days late will incur a penalty of 4^t %. Here t is rounded up to the nearest integer.

---

1. For sets A and B show that $A = (A \cap B) \cup (A \setminus B)$.
2. For sets B and $A_1, A_2, \ldots, A_n$ show that $B \cap \left(\bigcup_{i=1}^{n} A_i \right) = \bigcup_{i=1}^{n} (B \cap A_i)$.
3. Find an example of a collection of subsets A_i, $i \in \mathbb{N}$, with the property that each $A_i \subset \mathbb{N}$, $A_i \neq A_j$ for $i \neq j$, and $\bigcap_{i=1}^{\infty} A_i = \emptyset$.
4. Suppose $f : A \to B$ is injective. Show that there exists a bijection between A and its direct image under f.
5. Suppose $g : A \to C$ and $h : B \to C$. If h is bijective show that there exists a function $f : A \to B$ such that $g = h \circ f$.
6. Let A be a nonempty set and let F be the set of all functions that map A to A. Suppose that for every f and g in F
$$(f \circ g)(x) = (g \circ f)(x) \text{ for every } x \in A .$$
Show that A has only one element.
7. Let f and g be functions and suppose that $(g \circ f)(x) = x$ for every x in the domain of f. Show that f is injective and that the range of f is a subset of the domain of g.
8. Textbook exercise 0.3.8.
9. Give an example of a function f and sets A, X and Y such that $A \subset X$, $f : X \to Y$, yet $f^{-1}(f(A)) \neq A$.
10. Let $f : A \setminus B \to B \setminus A$ be a bijection. Find a bijection $g : A \to B$ and prove your result.