Math 371 - Introductory Real Analysis

Sep 6 2019

0.3: Set Theory

Sets

- **Definition:** A set S is a collection of objects called members or elements.
- Write $x \in S$ if x is an element of S, $x \notin S$ if x is not an element of S.
- $S = \emptyset$ if S contains no elements.

Subsets

- **Definition:** A set A is a subset of B, written $A \subset B$, if $x \in B$ for every $x \in A$.

  We write $A = B$ if both $A \subset B$ and $B \subset A$.

- Note that $A \subset B$ if $A = B$. To emphasize that $A \subset B$ but $A \neq B$ write $A \subsetneq B$.

Union, Intersection, Complement

Definition:

- Set union: $A \cup B = \{x : x \in A \text{ \underline{or} } x \in B\}$

- Set intersection: $A \cap B = \{x : x \in A \text{ \underline{and} } x \in B\}$

- Set complement: $A \setminus B = \{x : x \in A \text{ \underline{and} } x \notin B\}$

- If A is the universe, that is, the set of all elements under consideration, $A \setminus B$ can instead be written B^c, read "B complement".

Some Important Sets

- Natural numbers: $\mathbb{N} = \{1, 2, 3, \ldots\}$
- Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Rational numbers: $\mathbb{Q} = \left\{ \frac{p}{q} \ : \ p, q \in \mathbb{Z}, q \neq 0 \right\}$
- Real numbers: $\mathbb{R}$. More on the definition of $\mathbb{R}$ later.

Well Ordering Property of $\mathbb{N}$

We will assume the well Ordering Property of $\mathbb{N}$:

Every non-empty subset of $\mathbb{N}$ has a smallest element.

Fancier Unions and Intersections

Definition: Let $\mathcal{I}$ be a set, called an index set.

- $x \in \bigcup_{\alpha \in \mathcal{I}} A_\alpha$ if $x \in A_\beta$ for at least one $\beta \in \mathcal{I}$
- $x \in \bigcap_{\alpha \in \mathcal{I}} A_\alpha$ if $x \in A_\beta$ for every $\beta \in \mathcal{I}$