Review Problems 1: Sol'ns.

(1) $\vec{a} = \langle 1,1,-2\rangle$, $\vec{b} = \langle 3,-2,1\rangle$, $\vec{c} = \langle 0,1,-5\rangle$.

(a) $2\vec{a} + 3\vec{b} = 2\langle 1,1,-2\rangle + 3\langle 3,-2,1\rangle = \boxed{\langle 11,-4,-1\rangle}$

(b) $|b| = \sqrt{3^2+(-2)^2+1^2} = \boxed{\sqrt{14}}$

(c) $\vec{a}\cdot\vec{b} = (1)(3)+(1)(-2)+(-2)(1) = \boxed{-1}$

(d) $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ 3 & -2 & 1 \end{vmatrix} = \boxed{\langle -3,-7,-5\rangle}$

(e) $|\vec{b}\times\vec{c}| = \left| \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -2 & 1 \\ 0 & 1 & -5 \end{vmatrix} \right| = |\langle 9,15,3\rangle| = \sqrt{9^2+15^2+3^2} = \boxed{3\sqrt{35}}$

(f) $\vec{a}\cdot(\vec{b}\times\vec{c}) = \langle 1,1,-2\rangle\cdot\langle 9,15,3\rangle = 9+15-6 = 18$ using (e),

or $\vec{a}\cdot(\vec{b}\times\vec{c}) = \begin{vmatrix} 1 & 1 & -2 \\ 3 & -2 & 1 \\ 0 & 1 & -5 \end{vmatrix} = (1)(9)-(1)(-15)+(-2)(3) = \boxed{18}$

(g) $\vec{c}\times\vec{c} = \boxed{0}$

(h) $\vec{a}\times(\vec{b}\times\vec{c}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ 9 & 15 & 3 \end{vmatrix} = \boxed{\langle 33,-21,6\rangle}$

(i) $\text{comp}_{\vec{a}}\vec{b} = \vec{b}\cdot\dfrac{\vec{a}}{|\vec{a}|} = \dfrac{-1}{\sqrt{1^2+1^2+(-2)^2}} = \boxed{\dfrac{-1}{\sqrt{6}}}$

(j) $\text{proj}_{\vec{a}}\vec{b} = \text{comp}_{\vec{a}}\vec{b}\left(\dfrac{\vec{a}}{|\vec{a}|}\right) = \dfrac{-1}{\sqrt{6}}\,\dfrac{\langle 1,1,-2\rangle}{\sqrt{6}} = \boxed{\left\langle -\frac{1}{6}, -\frac{1}{6}, \frac{1}{3}\right\rangle}$

(k) $\theta = \arccos\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right) = \arccos\left(\dfrac{-1}{\sqrt{6}\sqrt{14}}\right) = \arccos\left(\dfrac{-1}{2\sqrt{21}}\right) \approx \boxed{96^\circ}$

(2.) Find abs. min. of $|\vec{r}(t)| = \sqrt{(1+t)^2+(2-t)^2+(-1+2t)^2}$. Value of t which minimizes $|\vec{r}(t)|$ is that which minimizes

$f(t) = |\vec{r}(t)|^2 = (1+t)^2+(2-t)^2+(-1+2t)^2 = 6t^2-6t+6$

$f'(t) = 12t-6 = 0$ at $t=\frac{1}{2}$ which must correspond to the abs. min. since f has an abs. min, and it must occur at a critical point. $\therefore$ Min. distance to line is $|\vec{r}(\tfrac{1}{2})| = \boxed{\dfrac{3}{\sqrt{2}}}$.

Review Problems 2: Sol^{n}s

(3)(a) Normal is $\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -2 & 9 \\ -1 & 2 & -5 \end{vmatrix} = \langle -8, -24, -8 \rangle = -8\underbrace{\langle 1,3,1 \rangle}_{\text{use } \langle 1,3,1\rangle \text{ as normal } \vec{n}}$

Using point $A(2,1,1)$: $\langle x-2, y-1, z-1 \rangle \cdot \langle 1,3,1 \rangle = 0$

$$\Rightarrow (x-2) + 3(y-1) + (z-1) = 0$$

$$\Rightarrow \boxed{x + 3y + z = 6}$$

(b) Using point $B(-1,-1,10)$ and direction vector $\vec{n} = \langle 1,3,1 \rangle$, line is

$\vec{r}(t) = \langle -1+t, -1+3t, 10+t \rangle$

$\therefore$ $x = -1+t \Rightarrow t = x+1$

$y = -1+3t \Rightarrow t = \frac{y+1}{3}$

$z = 10+t \Rightarrow t = z-10.$

Symmetric equations are $\boxed{x+1 = \frac{y+1}{3} = z-10}$

(c) The angle between planes is the angle between normals to planes : $\vec{n}_1 = \langle 1,3,1 \rangle$, $\vec{n}_2 = \langle 2,-4,-3 \rangle$;

$$\theta_1 = \arccos\left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1|\,|\vec{n}_2|}\right) = \arccos\left(\frac{1\cdot 2 + 3(-4) + 1(-3)}{\sqrt{1^2+3^2+1^2}\cdot\sqrt{2^2+(-4)^2+(-3)^2}}\right)$$

$$= \arccos\left(\frac{-13}{\sqrt{11}\sqrt{29}}\right) \approx 136.7^\circ$$

The acute angle θ is then $180^\circ - \theta_1 = \boxed{43.3^\circ}$

(d) Equation of 2^{nd} plane using $P(2,0,4)$ and $\vec{n}_2 = \langle 2,-4,-3 \rangle$

is $\langle x-2, y, z-4 \rangle \cdot \langle 2,-4,-3 \rangle = 0$

$$\Rightarrow 2(x-2) - 4y - 3(z-4) = 0$$

$$\Rightarrow 2x - 4y - 3z = -8$$

Line of intersection is solution to system:

① $x + 3y + z = 6$

② $2x - 4y - 3z = -8$

② − 2①: $-10y - 5z = -20$

$\Rightarrow$ ③ $2y + z = 4$

Letting $z = t$ in ③, $y = \frac{1}{2}(4-t) \Rightarrow x = 6 - \frac{3}{2}(4-t) - t = \frac{1}{2}t$ from ①.

$\therefore$ Line is $\vec{r}(t) = \langle \frac{1}{2}t, \frac{1}{2}(4-t), t \rangle$, or replacing t with $2t$:

$$\boxed{\vec{r}(t) = \langle t, 2-t, 2t \rangle}$$

Review Problem 1 : Sol^{n}s.

(4) (a) $x = 2 - t^3,\ y = 2t - 1,\ z = \ln(t)$.

Curve intersect xz-plane when $y = 0 \Rightarrow t = \frac{1}{2}$.

$\therefore$ point is $\left(2 - (\tfrac{1}{2})^3,\ 2(\tfrac{1}{2}) - 1,\ \ln(\tfrac{1}{2})\right) = \boxed{\left(\tfrac{15}{8}, 0, \ln(1/2)\right)}$

(b) At $(1,1,0)$ $t = 1$.

Tangent vector at $(1,1,0)$ is $\frac{d}{dt}\langle 2 - t^3, 2t - 1, \ln(t)\rangle\Big|_{t=1} = \langle -3, 2, 1\rangle$

$\therefore$ tangent line is $\boxed{\vec{r}(t) = \langle 1,1,0\rangle + t\langle -3,2,1\rangle}$

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(5) $\vec{s}(0) = \langle 0,0,0\rangle$; $\vec{s}\,'(0) = \langle 1,-1,3\rangle$, $\vec{s}\,''(t) = \langle 6t, 12t^2, -6t\rangle$.

$\therefore \vec{s}\,'(t) = \int \vec{s}\,''(t)\,dt = \langle 3t^2, 4t^3, -3t^2\rangle + \vec{c}_1$; $s'(0) = \langle 1,-1,3\rangle \Rightarrow \vec{c}_1 = \langle 1,-1,3\rangle$.

$\therefore \vec{s}\,'(t) = \langle 3t^2+1, 4t^3-1, -3t^2+3\rangle$

$\therefore \vec{s}(t) = \int \vec{s}\,'(t)\,dt = \langle t^3+t, t^4-t, -t^3+3t\rangle + \vec{c}_2$; $\vec{s}(0) = \vec{0} \Rightarrow \vec{c}_2 = \vec{0}$.

$\therefore \boxed{\vec{s}(t) = \langle t^3+t, t^4-t, -t^3+3t\rangle}$

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(6) $v = r\cos(s + 2t)$

$v_r = \cos(s+2t)$, $v_s = -r\sin(s+2t)$, $v_t = -2r\sin(s+2t)$

$v_{rr} = 0$; $v_{rs} = v_{sr} = -\sin(s+2t)$; $v_{rt} = v_{tr} = -2\sin(s+2t)$

$v_{st} = v_{ts} = -2r\cos(s+2t)$; $v_{ss} = -r\cos(s+2t)$

$v_{tt} = -4r\cos(s+2t)$

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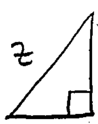
(7)



z

h = 12 ± 0.002 m

b = 5 ± 0.002 m

(a) $A = \frac{1}{2}bh$.

$$dA = \frac{\partial A}{\partial b}db + \frac{\partial A}{\partial h}dh$$
$$= \left(\frac{h}{2}\right)db + \left(\frac{b}{2}\right)dh$$
$$= \left(\frac{12}{2}\right)(0.002) + \left(\frac{5}{2}\right)(0.002)$$
$$= \boxed{0.017\ m^2}$$

(b) $z = (b^2+h^2)^{1/2}$

$$dz = \frac{\partial z}{\partial b}db + \frac{\partial z}{\partial h}dh$$
$$= (b^2+h^2)^{-\frac{1}{2}}\cdot b\cdot db + (b^2+h^2)^{-\frac{1}{2}}\cdot h\cdot dh$$

$$= (5^2+12^2)^{-\frac{1}{2}}\left[(5)(0.002) + (12)(0.002)\right]$$
$$= \frac{0.034}{13} \doteq \boxed{0.0026\ m}$$

Review Problems I: Sol^{n}s

(8)(a) $D_{\vec{u}}f$ is a maximum when ∇f points in the same direction as $\vec{u}$.

(b) $D_{\vec{u}}f$ is a minimum when ∇f points in the opposite direction to $\vec{u}$.

(c) $D_{\vec{u}}f = 0$ when $\nabla f \perp \vec{u}$ or $\nabla f = 0$.

(d) $D_{\vec{u}}f = \frac{1}{2}|\nabla f||\vec{u}| \Rightarrow \nabla f \cdot \vec{u} = \frac{1}{2}|\nabla f|\,|\nabla \vec{u}|$

$\Rightarrow \cancel{|\nabla f|}\cancel{|\vec{u}|}\cos\theta = \frac{1}{2}\cancel{|\nabla f|}\,\cancel{|\vec{u}|}$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \theta = \frac{\pi}{3}$

(9) Parametrization of curve of intersection :

$z = 2x^2 - y^2$ & $z = 4 \Rightarrow 2x^2 - y^2 = 4 \Rightarrow y^2 = 2x^2 - 4 \Rightarrow x = \pm\sqrt{\frac{1}{2}y^2 + 2}$

Letting $y = t$, $\vec{r}_1(t) = \langle \sqrt{\frac{1}{2}t^2+2}, t, 4 \rangle$, $\vec{r}_2 = \langle -\sqrt{\frac{1}{2}t^2+2}, t, 4 \rangle$.

$P(-2,2,4) = \vec{r}_2(2)$.

Direction vector of tangent line is $\vec{v} = \vec{r}_2\,'(2) = \left\langle \frac{-t}{2\sqrt{\frac{1}{2}t^2+2}}, 1, 0 \right\rangle \Big|_{t=2}$

$= \langle -\frac{1}{2}, 1, 0 \rangle$

$\therefore$ Tangent line is $\vec{r}(t) = \langle -2, 2, 4 \rangle + t\langle -\frac{1}{2}, 1, 0 \rangle$

or $\boxed{x = -2 - \frac{t}{2},\ y = 2 + t,\ z = 4}$

(10) $f(x,y) = (x^2+y)e^{y/2}$

$f_x = 2xe^{y/2}$; $f_y = e^{y/2} + \frac{1}{2}(x^2+y)e^{y/2} = (1 + \frac{1}{2}x^2 + \frac{1}{2}y)e^{y/2}$

$f_{xx} = 2e^{y/2}$; $f_{yy} = \frac{1}{2}e^{y/2} + \frac{1}{2}(1 + \frac{1}{2}x^2 + \frac{1}{2}y)e^{y/2} = (1 + \frac{1}{4}x^2 + \frac{1}{4}y)e^{y/2}$; $f_{xy} = xe^{y/2}$

$f_x = f_y = 0 \Rightarrow \begin{cases} 2xe^{y/2} = 0 \\ (1 + \frac{1}{2}x^2 + \frac{1}{2}y)e^{y/2} = 0 \end{cases} \Rightarrow x = 0,\ y = -2$

$f_{xx}(0,-2)f_{yy}(0,-2) - [f_{xy}(0,-2)]^2 = (2e^{-\frac{2}{2}})(1 + \frac{1}{4}(0)^2 + \frac{1}{4}(-2))e^{-\frac{2}{2}} - 0^2 > 0$,

$f_{xx}(0,-2) = 2e^{-2/2} > 0$; $\therefore (0,-2)$ is the only C.P. and corresponds to a loc. min. of $f(0,-2) = -2e^{-1}$.

Review Problems 1: sol^{n}s.

(11) $f(x,y) = e^{-(x^2+y^2)}(x^2+2y^2)$ on $D: \{(x,y) \mid x^2+y^2 \le 4\}$.

- C.P.'s of f in D:

$f_x = -2x\,e^{-(x^2+y^2)}(x^2+2y^2) + e^{-(x^2+y^2)}(2x) = e^{-(x^2+y^2)}(2x-2x^3-4xy^2)$

$f_y = -2y\,e^{-(x^2+y^2)}(x^2+2y^2) + e^{-(x^2+y^2)}(4y) = e^{-(x^2+y^2)}(-2x^2y+4y-4y^3)$

$$f_x = f_y = 0 \Rightarrow \begin{cases} 2x-2x^3-4xy^2 = 0 & ① \\ -2x^2y+4y-4y^3 = 0 & ② \end{cases}$$

$$\Rightarrow \begin{cases} 2x(1-x^2-2y^2) = 0 & ④ \\ 2y(2-2y^2-x^2) = 0 & ⑤ \end{cases}$$

④ $\Rightarrow$ $x=0$ or $x^2 = 1-2y^2$.

Sub. $x=0$ into ⑤ $\Rightarrow$ $y=0$ or $y=\pm1$: $(0,0),(0,1),(0,-1)$ are C.P.s.

sub. $x^2=1-2y^2$ into ⑤ $\Rightarrow$ $y=0$ $\Rightarrow$ $x^2=1 \Rightarrow x=\pm1$; $(1,0),(-1,0)$ are C.P.s.

Evaluating:

CP	$f(x,y)$
$(0,0)$	0
$(0,1)$	$2e^{-1}$
$(0,-1)$	$2e^{-1}$
$(1,0)$	e^{-1}
$(-1,0)$	e^{-1}

- On boundary of D: $x^2+y^2=4$,

$\therefore f(x,y) = g(y) = e^{-4}(4+y^2)$, $-2 \le y \le 2$

$g'(y) = e^{-4}(2y) = 0$ at $y=0$

Evaluating:

y	$g(y)$
-2	$8e^{-4}$
0	$4e^{-4}$
2	$8e^{-4}$

Selecting from two tables above:

f has abs. max of $2e^{-1}$, abs. min. of 0.