Question 1: Let $f(x,y) = \frac{1}{2}\ln(x^2+y^2) - e^{y/x}$. Calculate $f_x(1,0) + f_y(1,0)$

[3]

Question 2: Let $w(x,y,z) = 4x^3y^2z + \sec(x - \sqrt{1+z^2})$. Detemine w_{xzy}. [You may assume that Clairaut's Theorem applies.]

[3]

Question 3: Let $f(x,y,z) = xy + yz + xz$ where $x = \cos(u^2v)$, $y = \sin(v/u)$ and $z = u + v - 1$. Find $\frac{\partial f}{\partial u}$ at the point where $(u,v) = (1, \pi/2)$.

[4]

Question 4: Determine $\dfrac{\partial x}{\partial z}$ at the point $(x, y, z) = (1, -1, -3)$ if

$$xz + y\ln(x) - x^2 + 4 = 0$$

[4]

Question 5:

(i) Determine the directional derivative of $f(x, y) = x^2 e^{-2y}$ at the point $P(2, 0)$ in the direction of $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

[2]

(ii) If starting at the point $P(2, 0)$, in which direction in the xy-plane should one proceed so that $f(x, y)$ increases most rapidly? State your answer as a unit vector.

[2]

(iii) If starting at the point $P(2, 0)$, in which direction in the xy-plane should one proceed so that $f(x, y)$ neither increases nor decreases? State your answer as a unit vector. (There are two possible directions; give one.)

[2]

Question 6: Use a linear approximation to estimate $f(4.1, -0.2)$ where $f(x, y) = \sqrt{x + \sin(4y)}$.

[5]

Question 7: Find all points, if any, on the surface $z^2 - e^{xy} = 3$ at which the tangent plane is horizontal.

[5]

Question 8: Find all critical points of $f(x, y) = x^3 + 3xy + y^3$ and classify each as either a local maximum, local minimum or a saddle point. Carefully calculate all required derivatives and keep your work organized.

[5]

Question 9:
Find the volume of the largest rectangular box in the first octant having three faces on the coordinate planes and one vertex on the plane $x + 2y + 3z = 6$. You may assume that the solution you find corresponds to the maximum– there is no need to justify it.

[10]