Question 1: Eliminate the parameter to find a Cartesian equation of the following parametric curve, then sketch the curve for the given range of parameter values. Indicate with an arrow the direction in which the curve is traced as the t parameter increases:

$$x = \cos(t), \quad y = \sec(t), \quad 0 \le t < \pi/2$$

[5]

Question 2: For this question use the parametric curve $x = t^3 - 12t + 1$, $y = 2t - t^2$.

(i) Find an equation of the tangent line to the curve at the point where $t = -1$

[3]

(ii) Find all points (x, y) along the curve at which tangent lines are vertical.

[2]

Question 3: Determine the length of the parametric curve:

$$x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{3/2}}{3}, \quad 0 \leq t \leq 4$$

[5]

Question 4: Determine the area of the region between the parametric curve and the <u>y-axis</u>:

$$x = e^{-t} + 1, \quad y = e^{2t} - 1, \quad 0 \leq t \leq 1$$

[5]

Question 5: Neatly sketch the polar curve $r = 3\cos(2\theta)$.

[5]

Question 6: Find an equation (in Cartesian coordinates) for the tangent line to $r = 3\cos(\theta)$ at the point where $\theta = \pi/6$.

[5]

Question 7: Find the area enclosed by one loop of the polar curve $r = 3\cos(2\theta)$ from question 5.

[5]

Question 8: Find the arc length of $r = 3\cos(\theta)$ over $0 \leq \theta \leq \pi/6$.

[5]

Question 9: For this question use the sphere $x^2 + (y-3)^2 + (z+5)^2 = 4$:

(i) Find the point on the sphere that is closest to the xy-plane.

[2]

(ii) Find the point on the sphere that is closest to the point $(0, 7, -5)$.

[3]

Question 10: For this question use the vectors

$$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} \quad \text{and} \quad \mathbf{c} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

(i) Compute $|3\mathbf{a} - 2\mathbf{b} - \mathbf{c}|$.

[2]

(ii) Find a vector of magnitude 5 pointing in the same direction as $(\mathbf{b} - \mathbf{c})$.

[3]