Here are some additional problems that review material for the final exam. Questions from all major topics are included, but this is by no means an exhaustive list of all the material you should know for the final exam. It is intended as a further resource for exam preparation.

1. Sketch the graph of $f(x) = \begin{cases} x^2+1 & \text{if } x < -1 \\ 3x+1 & \text{if } x \geq -1 \end{cases}$ and find the following limits:
   (a) $\lim_{x\to -1^-} f(x)$
   (b) $\lim_{x\to -1^+} f(x)$
   (c) $\lim_{x\to -1} f(x)$
   (d) $\lim_{x\to 1} f(x)$

2. Evaluate the following limits:
   (a) $\displaystyle\lim_{x\to 0}(x^2-3x+1)$
   (b) $\displaystyle\lim_{x\to 3}\frac{x^2-x-6}{x-3}$
   (c) $\displaystyle\lim_{x\to 0}\frac{\sin x}{\tan x}$
   (d) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x}$
   (e) $\displaystyle\lim_{x\to 1}\frac{x-1}{\sqrt{x}-1}$
   (f) $\displaystyle\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$
   (g) $\displaystyle\lim_{t\to -1}\frac{\frac{1}{2}+\frac{1}{t}}{2+t}$
   (h) $\displaystyle\lim_{x\to 4^+}\sqrt{16-x^2}$
   (i) $\displaystyle\lim_{x\to -2^-}\sqrt{x^2+3x+2}$
   (j) $\displaystyle\lim_{x\to 1^-}\frac{1-2x}{x^2-1}$
   (k) $\displaystyle\lim_{x\to 1^+}\frac{1-2x}{x^2-1}$
   (l) $\displaystyle\lim_{x\to 1}\frac{1-2x}{x^2-1}$
   (m) $\displaystyle\lim_{x\to 2^+}\frac{x-4}{x^2-4x+4}$
   (n) $\displaystyle\lim_{x\to 2^-}\frac{-x}{\sqrt{4-x^2}}$
   (o) $\displaystyle\lim_{x\to -\infty}\frac{-x}{\sqrt{4+x^2}}$
   (p) $\displaystyle\lim_{x\to \infty}\frac{x^3-2}{3x^2+4x-1}$

3. Sketch the graph of a function that satisfies the following conditions:
   - $f(0)=0$
   - $f'(-2)=f'(1)=f'(9)=0$
   - $\lim_{x\to\infty} f(x)=0$
   - $\lim_{x\to 6} f(x)=-\infty$
   - $f'(x)<0$ on $(-\infty,-2)$, $(1,6)$ and $(9,\infty)$
   - $f'(x)>0$ on $(-2,1)$ and $(6,9)$
   - $f''(x)>0$ on $(-\infty,0)$ and $(12,\infty)$
   - $f''(x)<0$ on $(0,6)$ and $(6,12)$

4. Given that $\displaystyle\lim_{x\to 0^+}\frac{1-\cos x}{x^2}=\frac{1}{2}$, evaluate $\displaystyle\lim_{x\to 0^+}\frac{\sqrt{1-\cos x}}{x}$

5. Use the fact that $\lim_{x\to a} f(x)=2$, $\lim_{x\to a} g(x)=-3$ and $\lim_{x\to a} h(x)=0$ to find the following:
   (a) $\displaystyle\lim_{x\to a}[2f(x)-3g(x)]$
   (b) $\displaystyle\lim_{x\to a}\frac{f(x)+g(x)}{h(x)}$

6. Explain why each function is discontinuous at the given point.
   (a) $f(x)=\frac{x}{x-1}$ at $x=1$
   (b) $f(x)=\begin{cases} x^2 & \text{if } x<2 \\ 3 & \text{if } x=2 \\ 3x-2 & \text{if } x>2 \end{cases}$ at $x=2$.

7. On what intervals is $f(x)=\sqrt[3]{x+2}$ continuous?

8. Use the Intermediate Value Theorem to verify that $f(x)=x^2-7$ has a zero in the interval $[2,3]$.

9. Determine all of the horizontal and vertical asymptotes of the following functions.

(a) $f(x) = \frac{x}{4-x^2}$
(b) $y = \ln(1 - \cos x)$
(c) $f(x) = \frac{3x^2+1}{x^2-2x-3}$

10. Suppose that the length of a small animal t days after birth is $h(t) = \frac{300}{1+9(0.8)^t}$ mm. What is the length of the animal at birth? What is the eventual length of the animal?

11. Suppose that $f(x)$ is a rational function $f(x) = \frac{p(x)}{q(x)}$ with the degree of $p(x)$ greater than the degree of $q(x)$. Does $f(x)$ have a horizontal asymptote?

12. Use implicit differentiation to find $\frac{dy}{dx}$.

    (a) $x^2y^2 + 3y = 4x$
    (b) $\sqrt{xy} - 4y^2 = 12$
    (c) $\sin(xy) + x^2 = x - y$
    (d) $\frac{y}{x+1} - 3y = \tan x$
    (e) $\sqrt{x+y} - 4x^2 = y$

13. Use the definition of derivative to find

    (a) $f'(2)$ for $f(x) = x^2 - 2x$
    (b) $f'(x)$ for $f(x) = x + \sqrt{x}$
    (c) $f'(x)$ for $f(x) = \frac{x^2+1}{x-2}$

14. Find the derivatives of the following functions.

    (a) $y = x^4 - 3x^3 + 2x - 1$
    (b) $y = \frac{3}{\sqrt{x}} + \frac{5}{x^2}$
    (c) $y = t^2(t+2)^3$
    (d) $y = \frac{x}{3x^2-1}$
    (e) $y = x^2 \sin x$
    (f) $y = \tan(\sqrt{x})$
    (g) $y = \sqrt{\sin(4x)}$
    (h) $y = \left(\frac{x+1}{x-1}\right)^2$
    (i) $y = x^5\sqrt{x^3+2}$
    (j) $y = \frac{x^3}{(x^2+4)^2}$
    (k) $y = (\sqrt{x}+3)^{4/3}$
    (l) $y = (\sqrt{x^3+2} + 2x)^{-2}$
    (m) $y = \sqrt{\frac{x}{x^2+1}}$
    (n) $y = x\cos(5x^2)$
    (o) $y = \sin(\tan(x^2))$
    (p) $y = \frac{\sin(x^2)}{x^2}$
    (q) $y = \tan(\sqrt{x^2+1})$

15. Find an equation of the tangent line.

    (a) $y = x^4 - 2x + 1$ at $x = 1$
    (b) $y - x^2y^2 = x - 1$ at $(1, 1)$

16. Find the indicated derivative

    (a) $f''(x)$ for $f(x) = x^4 - 3x^3 + 2x^2 - x - 1$
    (b) $f''(x)$ for $f(x) = \tan x$
    (c) $f^{(26)}(x)$ for $f(x) = \sin(3x)$

17. Find all points at which the tangent line to the curve $y = x^3 - 6x^2 + 1$ is horizontal.

18. Let $f(t)$ represent the trading value of a stock at time t days. If $f'(t) < 0$, what does that mean about the stock?

19. Suppose f and g are differentiable with $f(0) = -1$, $f(1) = -2$, $f'(0) = -1$, $f'(1) = 3$, $g(0) = 3$, $g'(1) = 1$, $g'(0) = -1$ and $g'(1) = -2$.

    (a) Find an equation of the tangent line to $h(x) = f(x)g(x)$ at $x = 0$.
    (b) Find an equation of the tangent line to $h(x) = x^2f(x)$ at $x = 1$.

(c) Find $h'(-1)$ if $h(x) = f(x^4)$

(d) Find $h'(0)$ if $h(x) = \frac{g(x)}{f(x)}$

20. Suppose $s(t) = \sqrt{t^2 + 8}$ is the position function of some object, where t is measured in seconds and s is measured in meters. Find the velocity of the object at $t = 2$.

21. Oil spills out of a tanker at the rate of 120 gallons per minute. The oil spreads in a circle with a thickness of 0.25 inches. Given that 1 cubic foot equals 7.5 gallons, determine the rate at which the radius of the spill is increasing when the radius is 100 feet.

22. Sand is dumped such that the shape of the sandpile remains a cone with height equal to twice the radius. If the sand is dumped at the constant rate of 20 cubic feet per second, find the rate at which the radius is increasing when the height reaches 6 feet.

23. Find the general antiderivative for the following:

(a) $f(x) = (3x^4 - 3x)$

(b) $f(x) = \left(\frac{x^{1/3} - 3}{x^{2/3}}\right)$

(c) $f(x) = (2\sin x + \cos x)$

(d) $f(x) = 5\sec^2 x$

(e) $f(x) = x^{1/4}(x^{5/4} - 4)$

(f) $f(x) = \frac{e^x + 3}{e^x}$

24. Find the function $f(x)$ satisfying $f''(x) = 12$, $f'(0) = 2$ and $f(0) = 3$.

25. Determine the position function if the velocity function is $v(t) = 3 - 12t$ and the initial position is $s(0) = 3$.

26. Determine the position function if the acceleration function is $a(t) = 3\sin t + 1$, the initial velocity is $v(0) = 0$ and the initial position is $s(0) = 4$.

27. Find a function $f(x)$ such that the point $(1, 2)$ is on the graph of $f(x)$, the slope of the tangent line at $(1, 2)$ is 3 and $f''(x) = x - 1$.

28. Find the linear approximation to $f(x) = \sqrt{2x + 9}$ at $x = 0$.

29. Use linear approximations to estimate $\sqrt[4]{16.04}$

30. Find the critical numbers of the following functions.

(a) $f(x) = x^4 - 3x^3 + 2$

(b) $f(x) = x^{3/4} - 4x^{1/4}$

(c) $f(x) = \frac{x^2 - 2}{x + 2}$

(d) $f(x) = x^{4/3} + 4x^{1/3} + 4x^{-2/3}$

(e) $f(x) = 2x\sqrt{x + 1}$

(f) $f(x) = \frac{1}{2}(e^x + e^{-x})$

31. Find the absolute extrema of the given function on the given interval.

(a) $f(x) = x^3 - 3x + 1$ on $[0, 2]$

(b) $f(x) = x^{2/3}$ on $[-4, -2]$

(c) $f(x) = \frac{3x^2}{x - 3}$ on $[-2, 2]$

(d) $f(x) = e^{-x^2}$ on $[-3, 2]$

32. Sketch a graph of a function f such that the absolute maximum of $f(x)$ on $[-2, 2]$ is 3 and the absolute minimum does not exist.

33. Show that $f(x) = x^3 + bx^2 + cx + d$ has both a local minimum and a local maximum if $c < 0$.

34. For the family of functions $f(x) = x^4 + cx^3 + 1$ find the x coordinates of all local extrema. (Your answer will depend on c.)

35. Find the intervals where the function is increasing and decreasing.

    (a) $y = x^4 - 8x^2 + 1$
    (b) $y = (x+1)^{2/3}$
    (c) $y = \sin x + \cos x$
    (d) $y = e^{x^2-1}$

36. Find all critical numbers and use the First Derivative Test to classify each as a local maximum or local minimum.

    (a) $y = x^4 + 4x^3 - 2$
    (b) $y = \frac{x}{1+x^3}$
    (c) $y = \sqrt{x^3 + 3x^2}$
    (d) $y = xe^{-2x}$

37. Find all asymptotes and extrema of $y = \frac{x^2}{x^2-4x+3}$.

38. Sketch a graph of a function with the given properties: $f(3) = 0$, $f'(x) < 0$ for $x < 0$ and $x > 3$, $f'(x) > 0$ for $0 < x < 3$, $f'(3) = 0$, $f(0)$ and $f'(0)$ do not exist.

39. Suppose that the total sales of a product after t months is given by $s(t) = \sqrt{t+4}$ thousand dollars. Compute and interpret $s'(t)$.

40. Show that $f(x) = x^3 + bx^2 + cx + d$ is an increasing function if $b^2 \leq 3c$.

41. Determine the intervals where the function is concave up and concave down.

    (a) $f(x) = x + \frac{1}{x}$
    (b) $f(x) = \sin x - \cos x$
    (c) $f(x) = x^{4/3} + 4x^{1/3}$
    (d) $f(x) = xe^{-4x}$

42. Find all critical number and use the Second Derivative Test to determine all local extrema.

    (a) $f(x) = x^4 + 4x^3 - 1$
    (b) $f(x) = \frac{x^2-5x+4}{x}$
    (c) $f(x) = xe^{-x}$

43. Sketch a graph with the following properties: $f(0) = 0$, $f(-1) = 1$, $f(1) = 1$, $f'(x) > 0$ for $x < -1$ and $0 < x < 1$, $f'(x) < 0$ for $-1 < x < 0$ and $x > 1$, $f''(x) < 0$ for $x < 0$ and $x > 0$.

44. Suppose $w(t)$ is the depth of water in a city's water reservoir at time t. Which would be better news at time $t = 0$, $w''(0) = 0.05$ or $w''(0) = -0.05$ or would you need to know that value of $w'(0)$ to determine which is better?

45. A three sided fence is to be built next to a straight section of river which forms the fourth side of a rectangular region. The enclosed area is to equal 1800 square feet. Find the minimum perimeter and the dimensions of the corresponding enclosure.

46. A two-pen corral is to be built. The outline of the corral forms two identical adjoining rectangles. If there is 120 feet of fencing available, what dimensions of the corral will maximize the enclosed area?

47. Find the point on the curve $y = x^2$ closest to the point $(0, 1)$.

48. Find the point on the curve $y = \cos x$ closest to the point $(0, 0)$.

49. A company's revenue for selling x (thousand) items is given by $R(x) = \frac{35x-x^2}{x^2+35}$. Find the value of x that maximizes the revenue and find the maximum revenue.

50. Completely analyze each function (according to the checklist for curve sketching) and then use the information collected to graph the function.

    (a) $f(x) = x^5 - 2x^3 + 1$
    (b) $f(x) = x + \frac{4}{x}$
    (c) $f(x) = \sqrt{x^2 + 1}$
    (d) $f(x) = \sqrt[3]{x^3 - 3x^2 + 2x}$
    (e) $f(x) = (x^3 - 3x^2 + 2x)^{2/3}$
    (f) $f(x) = \frac{x^2+1}{3x^2-1}$
    (g) $f(x) = \frac{5x}{x^3-x+1}$

51. The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error in computing

    (a) the volume of the cube
    (b) the surface area of the cube

52. Find two positive numbers whose product is 100 and whose sum is a minimum.

53. Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

54. A balloon is rising at a constant speed of 5ft/s. A boy is cycling along a straight road at a speed of 15ft/s. When he passes under the balloon it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 seconds later?

55. Find a parabola $y = ax^2 + bx + c$ that passes through the point $(1, 4)$ and whose tangent lines at $x = -1$ and $x = 5$ have slopes 6 and -2 respectively.

56. Sketch the graph of a function f that is continuous on $[1, 5]$ and has the following properties:

    - absolute maximum at 5
    - absolute minimum at 2
    - local maximum at 3
    - local minima at 2 and 4

57. An airplane flying west at 300 miles per hour goes over the control tower at noon. A second airplane at the the same altitude flying north at 400 miles per hour goes over the control tower one hour later. How fast is the distance between the airplanes changing at 2:00 pm?

58. Chris, who is 6 feet tall, is walking away from a street light pole 30 feet high at a rate of 2 feet per second.

    (a) How fast is his shadow increasing in length when Chris is 24 feet from the pole?
    (b) How fast is the tip of his shadow moving?

59. Find the dimensions of an aluminum can that holds 40 cubic centimeters of water and that uses the least amount of material. Assume the can is a cylinder with a top and a bottom.

60. Show that $f(x) = 2x^3 + 1$ and $g(x) = \sqrt[3]{\frac{x-1}{2}}$ are inverse functions.

61. Determine if the function is one-to-one. If it is, find the inverse function.

(a) $f(x) = \sqrt{x-1}$
(b) $f(x) = \frac{1}{x+1}$
(c) $f(x) = \frac{1-2x}{1+x}$
(d) $f(x) = x^4 + 2$
(e) $f(x) = \sqrt{x^3+1}$

62. Solve the following equations for x.

(a) $\log_4(x+4) - 2\log_4(x+1) = \frac{1}{2}$
(b) $2^{x^2-1} = 32$
(c) $\log_2(x+3) - \log_2 x = 2$
(d) $e^x(x^2-1) = 0$
(e) $xe^{-2x} + 2e^{-2x} = 0$
(f) $4\ln x = -8$
(g) $x^2 \ln x - 9\ln x = 0$
(h) $e^{2\ln x} = 4$

63. Rewrite $\log_a(x^4+3x^2+2) + \log_a(x^4+5x^2+6) - 4\log_a\sqrt{x^2+2}$ as a single logarithm.

64. Assume that the given function has an inverse. Without solving for the inverse, find the indicated function values.

(a) $f(x) = x^3 + 4x - 1$; $f^{-1}(-1)$ and $f^{-1}(4)$
(b) $f(x) = \sqrt{x^3+2x+4}$; $f^{-1}(4)$ and $f^{-1}(2)$

65. Evaluate the following limits.

(a) $\displaystyle\lim_{x\to 0} \frac{xe^{-2x+1}}{x^2+x}$
(b) $\displaystyle\lim_{x\to 0} \frac{1-e^{2x}}{1-e^x}$
(c) $\displaystyle\lim_{x\to\infty} \ln(2x)$
(d) $\displaystyle\lim_{x\to 0^+} e^{-2/x}$
(e) $\displaystyle\lim_{x\to\infty} e^{2x-1}$

66. Differentiate the following functions.

(a) $y = e^{2x}\cos(4x)$
(b) $y = x4^{3x}$
(c) $y = 2e^{4x+1}$
(d) $y = 4^{-3x+1}$
(e) $y = \frac{x}{e^{6x}}$
(f) $y = \ln(x^3+x)$
(g) $y = x^3\ln x$
(h) $y = \ln(\cos x)$
(i) $y = e^{\sin(2x)}$
(j) $y = \sin(\ln(\cos x^3))$
(k) $y = \ln(\sin x^2)$
(l) $y = \frac{\sqrt{\ln x^2}}{x}$
(m) $y = \ln(\sec x + \tan x)$
(n) $y = \sqrt[3]{e^{2x}x^3}$

67. Use logarithmic differentiation to find $\frac{dy}{dx}$ for to following.

(a) $y = \dfrac{\sqrt{x+13}}{(x-4)\sqrt[3]{2x+1}}$
(b) $y = x^{\sin x}$
(c) $y = (x^2)^{4x}$
(d) $y = x^{\ln x}$
(e) $y = x^{\sqrt{x}}$

68. Differentiate the following functions.

(a) $y = \arcsin(2x^2)$
(b) $y = x^3 \arctan(e^x)$
(c) $y = e^x \arcsin(x^2)$
(d) $y = \tan(\arccos x)$
(e) $y = \arctan(\ln x)$
(f) $y = \frac{\arctan x}{(1+x^2)^2}$

69. Find an expression for each of the following in terms of x.

(a) $\cos(\arcsin x)$
(b) $\cos(\arctan x)$

70. Use implicit differentiation to find $\frac{dy}{dx}$ for the following equations.

(a) $\arctan x + \arctan y = \frac{\pi}{2}$
(b) $(\arcsin x)(\arcsin y) = \frac{\pi^2}{16}$

Answers

1. (a) 2 (b) -2 (c) Does not exist (d) 4

2. (a) 1 (b) 5 (c) 1 (d) $\frac{1}{4}$ (e) 2 (f) $\frac{1}{2}$ (g) $-\frac{1}{2}$ (h) Does not exist (i) 0 (j) ∞ (k) $-\infty$ (l) does not exist (m) $-\infty$ (n) $-\infty$ (o) 1 (p) ∞

3. Answers will vary

4. $\frac{1}{\sqrt{2}}$

5. (a) 13 (b) Does not exist

6. (a) $f(1)$ is not defined (b) $\lim_{x\to 2} f(x) \neq f(2)$

7. $(-\infty, \infty)$

8. Observe that f is continuous on $[2, 3]$ with $f(2) < 0$, $f(3) > 0$, so by the Intermediate Value theorem $f(c) = 0$ for some $2 < c < 3$.

9. (a) $x = \pm 2$, $y = 0$ (b) $x = 2\pi n$ where n is an integer (c) $x = -1$, $x = 3$, $y = 3$

10. 30mm, 300mm

11. no

12. (a) $\frac{4-2xy^2}{3+2x^2y}$ (b) $\frac{y}{16y\sqrt{xy}-x}$ (c) $\frac{1-2x-y\cos(xy)}{x\cos(xy)+1}$ (d) $\frac{\sec^2 x+\frac{y}{(x+1)^2}}{\frac{1}{(x+1)}-3}$ (e) $\frac{16x\sqrt{x+y}-1}{1+2\sqrt{x+y}}$

13. (a) 2 (b) $1+\frac{1}{2}x^{-\frac{1}{2}}$ (c) $\frac{x^2-4x-1}{(x-2)^2}$

14. (a) $4x^3 - 9x^2 + 2$
(b) $-\frac{3}{2}x^{-3/2} - 10x^{-3}$
(c) $2t(t+2)^3 + 3t^2(t+2)^2$
(d) $\frac{(3x^2-1)-6x^2}{(3x^2-1)^2}$
(e) $2x\sin x + x^2\cos x$
(f) $\frac{1}{2}x^{-1/2}\sec^2(\sqrt{x})$
(g) $2\cos(4x)(\sin(4x))^{-1/2}$
(h) $\frac{-4(x+1)}{(x-1)^3}$
(i) $\frac{x^4(13x^3+20)}{2\sqrt{x^3+2}}$
(j) $\frac{-x^2(x^2-12)}{(x^2+4)^3}$
(k) $\frac{2(\sqrt{x}+3)^{1/3}}{3\sqrt{x}}$
(l) $\frac{-3x^2(x^3+2)^{-1/2}-4}{(\sqrt{x^3+2}+2x)^3}$
(m) $\frac{-x^2+1}{2\sqrt{x}(x^2+1)^{3/2}}$
(n) $\cos(5x^2) - 10x^2\sin(5x^2)$
(o) $2x\cos(\tan(x^2))\sec^2(x^2)$
(p) $2x^{-3}(x^2\cos(x^2) - \sin(x^2))$
(q) $\frac{x}{\sqrt{x^2+1}}\sec^2(\sqrt{x^2+1})$

15. (a) $y = 2x - 2$ (b) $y = -3(x-1) + 1$

16. (a) $12x^2 - 18x + 4$ (b) $2\sec^2 x \tan x$ (c) $-3^{26}\sin(3x)$

17. $(0, 1)$, $(4, -31)$

18. The stock is losing value.

19. (a) $y = -2x - 3$ (b) $y = -x - 1$ (c) -12 (d) -8

20. $\frac{1}{\sqrt{3}}$ meters per second

21. 1.22 feet per minute

22. $\frac{10}{9\pi}$ feet per second

23. (a) $\frac{3}{5}x^5 - \frac{3}{2}x^2 + C$ (c) $\frac{3}{2}x^{2/3} - 9x^{1/3} + C$ item $-2\cos x + \sin x + C$ (e) $\frac{2}{5}x^{5/2} - \frac{16}{5}x^{5/4} + C$
(b) $2x^{3/2} + \frac{1}{3}x^{-3} + C$ (d) $5\tan x + C$ (f) $x - 3e^{-x} + C$

24. $6x^2 + 2x + 3$

25. $3t - 6t^2 + 3$

26. $-3\sin t + \frac{1}{2}t^2 + 3t + 4$

27. $f(x) = \frac{x^3}{6} - \frac{x^2}{2} + \frac{7}{2}x - \frac{7}{6}$

28. $3 + \frac{1}{3}x$

29. 2.00125

30. (a) $0, \frac{9}{4}$ (c) $-2 + \sqrt{2}, -2 - \sqrt{2}$ (e) $\frac{-2}{3}, -1$
(b) $0, \frac{16}{9}$ (d) $-2, 1$ (f) 0

31. (a) minimum -1, maximum 3 (c) maximum 0, minimum -12
(b) maximum $2^{4/3}$, minimum $2^{2/3}$ (d) maximum 1, minimum e^{-9}

32. Answers will vary.

33. $4b^2 - 12c > 0$ if $c < 0$

34. there is a single relative minimum at $x = -3c/4$.

35. (a) increasing on $(-2, 0)$, $(2, \infty)$; decreasing on $(-infty, -2)$, $(0, 2)$
(b) increasing on $(-1, \infty)$; decreasing on $(-\infty, -1)$
(c) increasing on $(-\frac{3\pi}{4} + 2n\pi, \frac{\pi}{4} + 2n\pi)$; decreasing on $(\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi)$
(d) increasing on $(0, \infty)$; decreasing on $(-\infty, 0)$

36. (a) $x = -3$ (local minimum); $x = 0$ (neither)
    (b) $x = \frac{1}{\sqrt[3]{2}}$ (local maximum)
    (c) $x = -2$ (local maximum), $x = 0$ (local minimum)
    (d) $x = 1/2$ (local maximum)

37. vertical asymptotes at $x = 1$, $x = 3$, horizontal asymptote at $y = 1$, local minimum at $x = 0$, local maximum at $x = \frac{3}{2}$

38. Answers will vary

39. $s'(t) = \frac{1}{2\sqrt{t+4}}$; rate of increase of sales

40. Show that $f(x)$ is increasing everywhere

41. (a) concave up on $(0, \infty)$, concave down on $(-\infty, 0)$
    (b) concave up on $(-\frac{3\pi}{4} + 2n\pi, \frac{\pi}{4} + 2n\pi)$; concave down on $(\frac{\pi}{4} + 2n\pi, \frac{5\pi}{4} + 2n\pi)$
    (c) concave up on $(-\infty, 0)$, $(2, \infty)$, concave down on $(0, 2)$
    (d) concave up on $(-\infty, \frac{1}{2})$, concave down on $(\frac{1}{2}, \infty)$

42. (a) $x = -3$ (local minimum), $x = 0$ (test fails)
    (b) $x = -2$ (local maximum), $x = 2$ (local minimum)
    (c) $x = 1$ (local maximum)

43. Answers will vary

44. We need to know $w'(0)$

45. 30 feet by 60 feet; the perimeter is 120 feet

46. 20 feet by 30 feet

47. $(\frac{1}{\sqrt{2}}, \frac{1}{2})$ or $(-\frac{1}{\sqrt{2}}, \frac{1}{2})$

48. $(0, 1)$

49. $x = 5$, $R(5) = 2.5$

50. Graphs omitted

51. (a) 270 cm^3
    (b) 36 cm^2

52. 10, 10

53. 500, 125

54. 13 ft/s

55. $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

56. Answers will vary

57. 471 miles per hour

58. (a) 0.5 feet per second
    (b) 2.5 feet per second

59. The radius is 1.85 cm and the height is 3.7 cm

60. Show that $f(g(x)) = x$ and $g(f(x)) = x$

61. (a) $f^{-1}(x) = x^2 + 1$
    (b) $f^{-1}(x) = \frac{1}{x} - 1$
    (c) $f^{-1}(x) = \frac{1-x}{2+x}$
    (d) $f(x)$ is not one-to-one
    (e) $f^{-1}(x) = \sqrt[3]{x^2 - 1}$

62. (a) $\frac{1}{2}, -2$
    (b) $\pm\sqrt{6}$
    (c) 1
    (d) ± 1
    (e) -2
    (f) e^{-2}
    (g) $1, 3, -3$
    (h) ± 2

63. $\log_a(x^4 + 4x^2 + 3)$

64. (a) 0, 1
    (b) 2, 0

65. (a) e
    (b) 2
    (c) ∞
    (d) 0
    (e) ∞

66. (a) $2e^{2x}[\cos(4x) - 2\sin(4x)]$
    (b) $4^{3x} + 3x4^{3x} \ln 4$
    (c) $8e^{4x+1}$
    (d) $(-3\ln 4)4^{-3x+1}$
    (e) $\frac{e^{6x} - 6xe^{6x}}{e^{12x}}$
    (f) $\frac{3x^2+1}{x^3+x}$
    (g) $3x^2 \ln x + x^2$
    (h) $\frac{-\sin x}{\cos x}$
    (i) $2e^{\sin(2x)} \cos(2x)$
    (j) $\frac{-3x^2 \sin(x^3)\cos(\ln(\cos x^3))}{\cos x^3}$
    (k) $\frac{2x\cos x^2}{\sin x^2}$
    (l) $\frac{(\ln x^2)^{-1/2} - \sqrt{\ln x^2}}{x^2}$
    (m) $\frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$
    (n) $\frac{1}{3}(e^{2x}x^3)^{-2/3}[2x^3e^{2x} + 3x^2e^{2x}]$

67. (a) $-\frac{10x^2+219x-118}{6(x-4)^2(x+13)^{1/2}(2x+1)^{4/3}}$
    (b) $x^{\sin x}(\cos x \ln x + \frac{\sin x}{x})$
    (c) $(x^2)^{4x}(4\ln x^2 + 8)$
    (d) $x^{\ln x}\left(\frac{2\ln x}{x}\right)$
    (e) $x^{\sqrt{x}}(\frac{1}{2}x^{-1/2}\ln x + x^{-1/2})$

68. (a) $\frac{4x}{\sqrt{1-4x^4}}$
    (b) $x^2\left(\frac{xe^x}{1+e^{2x}} + 3\arctan(e^x)\right)$
    (c) $e^x \arcsin(x^2) + \frac{2xe^x}{\sqrt{1-x^4}}$
    (d) $-\frac{\sec^2(\arccos x)}{\sqrt{1-x^2}}$
    (e) $\frac{1}{x + x(\ln x)^2}$
    (f) $\frac{1-4x\arctan x}{(1+x^2)^3}$

69. (a) $\sqrt{1-x^2}$
    (b) $\frac{1}{\sqrt{1+x^2}}$

70. (a) $\frac{dy}{dx} = \frac{-1-y^2}{1+x^2}$
    (b) $\frac{dy}{dx} = \frac{-\arcsin x\sqrt{1-y^2}}{\arcsin x\sqrt{1-x^2}}$