

Math 121 Test 3 – Nov 27 2019

name (printed)

student number

Instructions:

1.

There are **7 pages** (including this cover page) in the test. You will be given **80 minutes** to write the test. Justify every answer, and clearly show your work. Unsupported answers will receive no credit. Read over the test before you begin.
2.

No notes or books are to be used during the test. The last page is for scrap work. Put your name on the scrap paper and return it along with your completed test. **To be considered for grading, your test must include the scrap paper with your name on it.**
3.

Calculators are not permitted. The use of a calculator, music player, cell phone or any other electronic device during the test is cheating.
4.

At the end of the test you will be given the instruction to stop writing. **Continuing to write after this instruction is cheating.**
5.

Academic dishonesty: Exposing your paper to another student, copying material from another student, or representing your work as that of another student constitutes academic dishonesty. Cases of academic dishonesty may lead to a zero grade in the test, a zero grade in the course, and other measures, such as suspension from this university.

question	value	score
1	10	
2	7	
3	3	
4	5	
5	5	
6	10	
7	5	
8	5	
Total	50	

Question 1: For this question use the fact that a sphere (ball) of radius r has volume $V = \frac{4}{3}\pi r^3$ and outer surface area $S = 4\pi r^2$.

An iron ball of radius 4 cm is coated with a layer of ice of uniform thickness and the ice is melting at a rate of $10 \text{ cm}^3/\text{min}$.

(a) How fast is the thickness of the ice decreasing when it is 2 cm thick?

[6]

(b) How fast is the outer surface area the ice decreasing when it is 2 cm thick?

[4]

Question 2: Find the following derivatives (Take care to use proper notation; it is not necessary to simplify your answers):

(a) $f(x) = \sqrt{x} [\ln(x)]^2$

[2]

(b) $y = 10e^{-5/x}$

[2]

(c) $g(t) = \log_2(4^{\sec(t)-t})$

[3]

Question 3: Find the slope of the tangent line to the curve $e^{y/x} = x^3y + x - 2$ at the point $(x, y) = (2, 0)$.

[3]

Question 4: Use a linear approximation to estimate $(2.2)^3$.

[5]

Question 5: Find an equation of the tangent line to $y = x^{\sin(x)}$ at the point where $x = \frac{\pi}{2}$.
[Hint: logarithmic differentiation]

[5]

Question 6: Evaluate the following limits, if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning.

(a) $\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x^2}$

[3]

(b) $\lim_{x \rightarrow \infty} \frac{e^x - e^{-x} + 2x}{x^2 + \ln(x)}$

[3]

(c) $\lim_{x \rightarrow 0^+} x[\ln(x)]^2$

[4]

Question 7: Determine the absolute minimum and maximum values of $f(x) = \frac{x}{x^2 + 1}$ on the interval $[-3, 4]$.

[5]

Question 8: Determine the intervals of increase and decrease of $f(x) = x^2 e^{-x}$.

[5]
