Question 1: Evaluate the following limits, if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate limits.)

(a) $\lim_{x\to 0} \dfrac{3x+1}{x^2(x+1)}$

[2]

(b) $\lim_{x\to -4} \dfrac{2x+8}{|x+4|}$

[4]

Question 2: Decide whether the following function is continuous at $x = 4$. Explain your reasoning.

$$f(x) = \begin{cases} \dfrac{x^2-4x}{x^2-3x-4} & \text{if } x \neq 4 \\ 5/4 & \text{if } x = 4 \end{cases}$$

[4]

Question 3:

(a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \sqrt{x+1}$. Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

[6]

(b) Find an equation of the tangent line to the graph of $y = \sqrt{x+1}$ at the point where $x = 8$.

[4]

Question 4: Find the following derivatives (it is not necessary to simplify your answers, but marks will be deducted for improper use of notation):

(a) $f(x) = \dfrac{\sqrt{x}}{1+\sqrt{x}}$

[2]

(b) $y = (t^3 - 3)(2\tan(t) + t)$

[3]

(c) $g(x) = \dfrac{\sec(x)}{1+x+x^2}$

[3]

(d) $y = 3\sin(\pi) - 2\cos(\pi) - \dfrac{\pi^2}{x}$

[2]

Question 5: Find the following derivatives (it is not necessary to simplify your answers, but marks will be deducted for improper use of notation):

(a) $y = (x^2 + \csc(x) + 1)^3$

[2]

(b) $y = \cos^4(1 - 2t)$

[3]

(c) $g(x) = \sqrt{\dfrac{x^3 + x}{x^5}}$

[3]

(d) $y = \sqrt[3]{x}\,[\sin(x) - \cos(x)]^2$

[2]

Question 6: A rock thrown vertically from the surface of the moon reaches a height of

$$s(t) = 24t - kt^2 \text{ metres}$$

in t seconds where k is a constant. The acceleration experienced by the rock is a constant -1.6 m/s^2. How long does it take for the rock to reach a velocity equal to half of its initial value?

[5]

Question 7: Find an equation of the tangent line to the curve defined by $y = 2\sin(x - y)$ at the point $(x, y) = (\pi, 0)$.

[5]