Question 1: Expand and simplify: $(1+2x)(x^2-3x+1)$

$$= x^2-3x+1+2x^3-6x^2+2x$$

$$= \boxed{2x^3-5x^2-x+1}$$

[3]

Question 2: Express as a single simplified fraction: $\frac{u+1}{1}+\frac{u}{u+1}$

$$= \frac{(u+1)(u+1)+u}{u+1}$$

$$= \frac{u^2+2u+1+u}{u+1}$$

$$= \boxed{\frac{u^2+3u+1}{u+1}}$$

[3]

Question 3: Simplify: $\left(\frac{\sqrt{xy}}{x^4}\right)\left(\frac{9y^{4/3}}{(3xy)^3}\right)$

$$= \frac{x^{1/2}y^{1/2}\cdot 9y^{4/3}}{x^4\cdot 3^3\cdot x^3\cdot y^3}$$

$$= \boxed{\frac{1}{3x^{13/2}y^{7/6}}}$$

[4]

Question 4: Express as a single simplified fraction: $\dfrac{x}{x^2-16} - \dfrac{x-2}{x^2+3x-4}$

$$= \frac{x}{(x-4)(x+4)} - \frac{x-2}{(x-1)(x+4)}$$

$$= \frac{x(x-1)-(x-2)(x-4)}{(x-4)(x+4)(x-1)}$$

$$= \frac{\cancel{x^2}-x-\cancel{x^2}+6x-8}{(x-4)(x+4)(x-1)}$$

$$= \boxed{\frac{5x-8}{(x-4)(x+4)(x-1)}}$$

[3]

Question 5: Rationalize and simplify: $\dfrac{\sqrt{x^2+x}-\sqrt{x^2-x}}{1} \cdot \dfrac{\sqrt{x^2+x}+\sqrt{x^2-x}}{\sqrt{x^2+x}+\sqrt{x^2-x}}$

$$= \frac{\cancel{x^2}+x-\cancel{x^2}+x}{\sqrt{x^2+x}+\sqrt{x^2-x}}$$

$$= \boxed{\frac{2x}{\sqrt{x^2+x}+\sqrt{x^2-x}}}$$

[4]

Question 6: The lines $ax+3y+p=0$ and $7x+by+q=0$ are perpendicular (here a, b, p, q are constants). Determine $\frac{a}{b}$.

$ax+3y+p=0 \Rightarrow y = -\frac{a}{3}x - \frac{p}{3}$ } Slope $m_1 = -\frac{a}{3}$

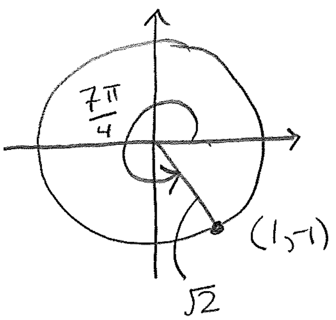
$7x+by+q=0 \Rightarrow y = -\frac{7}{b} - \frac{q}{b}$ } Slope $m_2 = -\frac{7}{b}$

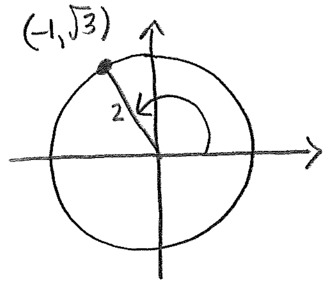
Since the lines are perpendicular $m_1 = \frac{-1}{m_2}$,

So $-\frac{a}{3} = \frac{-1}{\left(-\frac{7}{b}\right)} \Rightarrow -\frac{a}{3} = \frac{b}{7} \Rightarrow \frac{a}{b} = \boxed{-\frac{3}{7}}$

[3]

Question 7: Determine $\tan(7\pi/4) - \csc(2\pi/3)$. Express your answer as a single simplified fraction.





$$\tan\left(\frac{7\pi}{4}\right) - \csc\left(\frac{2\pi}{3}\right)$$

$$= \left(\frac{-1}{1}\right) - \left(\frac{2}{\sqrt{3}}\right)$$

$$= \frac{-\sqrt{3} - 2}{\sqrt{3}}$$

$$\text{or } \boxed{-\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)}$$

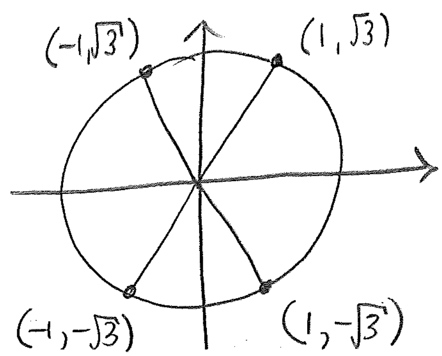
[3]

Question 8: Find all values of x in the interval $[0, 2\pi]$ for which $2\tan^2(x) - 1 = 5$.

$$2\tan^2 x - 1 = 5$$

$$\tan^2 x = \frac{5+1}{2} = 3$$

$$\text{So } \tan x = \pm\frac{\sqrt{3}}{1},$$



$$\boxed{\text{So } x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$$

[3]

Question 9: Let $f(x) = x + \dfrac{1}{x}$ and $g(x) = \dfrac{x+1}{x+2}$. Determine and simplify $(f \circ g)(x)$ and state the domain.

$$(f\circ g)(x) = f(g(x))$$

$$= \frac{x+1}{x+2} + \frac{1}{\left(\frac{x+1}{x+2}\right)} \quad \Big\} \; x \neq -2, \; x \neq 1$$

$$= \frac{x+1}{x+2} + \frac{x+2}{x+1}$$

$$= \frac{(x+1)^2 + (x+2)^2}{(x+1)(x+2)} = \boxed{\frac{2x^2+6x+5}{(x+1)(x+2)}, \text{ domain is } (-\infty,-2)\cup(-2,-1)\cup(-1,\infty)}$$

[4]

Question 10: Evaluate and simplify the difference quotient $\frac{f(a+h)-f(a)}{h}$ where $f(x) = \frac{x}{x+1}$. Express your answer as a single simplified fraction.

$$\frac{f(a+h)-f(a)}{h} = \frac{1}{h}\left[\frac{a+h}{a+h+1} - \frac{a}{a+1}\right]$$

$$= \frac{1}{h}\left[\frac{(a+h)(a+1) - a(a+h+1)}{(a+h+1)(a+1)}\right]$$

$$= \frac{1}{h}\left[\frac{\cancel{a^2}+\cancel{ah}+\cancel{a}+h-\cancel{a^2}-\cancel{ah}-\cancel{a}}{(a+h+1)(a+1)}\right]$$

$$= \frac{\cancel{h}}{\cancel{h}(a+h+1)(a+1)} = \boxed{\frac{1}{(a+h+1)(a+1)}}$$

[4]

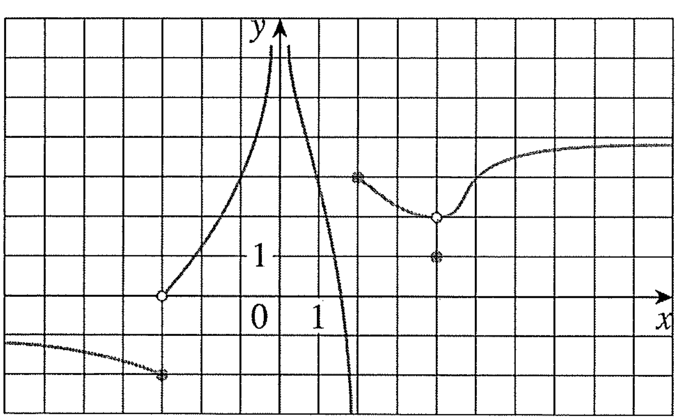
Question 11: Suppose $H(x) = \frac{1}{x+\sqrt{x}}$. Find functions $f(x)$ and $g(x)$ so that $H(x) = (f \circ g)(x)$. Do not let $f(x) = x$ or $g(x) = x$. (There are many possible correct answers.)

$\boxed{g(x) = x + \sqrt{x}, \quad f(x) = \frac{1}{x}}$

or $\boxed{g(x) = \sqrt{x}, \quad f(x) = \frac{1}{x^2+x}}$

[3]

Question 12: Consider the following graph of $y = f(x)$:



Let

$a = \lim_{x \to -3^-} f(x) = -2$

$b = \lim_{x \to 4} f(x) = 2$

and

$c = f(4) = 1$

Determine $a + b + c$.

$a+b+c = (-2)+2+1$

$= \boxed{1}$

[3]

Question 13: Evaluate the following limits, if they exist:

(a) $\lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h}$ $\rightarrow$ "$\frac{0}{0}$"

$= \lim_{h \to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2}$

$= \lim_{h \to 0} \frac{5\cancel{h}\cancel{+4}\cancel{-4}}{\cancel{h}(\sqrt{5h+4}+2)}$

$= \boxed{\frac{5}{4}}$

[4]

(b) $\lim_{x \to -2} \frac{x^2+x-2}{x^2+7x+10}$ $\} \rightarrow$ "$\frac{0}{0}$"

$= \lim_{x \to -2} \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x+5)}$

$= \frac{-3}{3}$

$= \boxed{-1}$

[3]

(c) $\lim_{x \to 1} \frac{\left(\frac{1}{x}-1\right)}{x-1}$ $\} \rightarrow$ "$\frac{0}{0}$"

$= \lim_{x \to 1} \left(\frac{1-x}{x}\right)\left(\frac{1}{x-1}\right)$

$= \lim_{x \to 1} -\left(\frac{\cancel{x-1}}{x}\right)\left(\frac{1}{\cancel{x-1}}\right)$

$= \boxed{-1}$

[3]