Curve Sketching

So far we have seen that

(i) If $f'(x) > 0$ on an interval then the graph of $y = f(x)$ is increasing on the interval;

(ii) If $f'(x) < 0$ on an interval then the graph of $y = f(x)$ is decreasing on the interval;

(iii) If $f''(x) > 0$ on an interval then the graph of $y = f(x)$ is concave up on the interval;

(iv) If $f''(x) < 0$ on an interval then the graph of $y = f(x)$ is concave down on the interval.

Using this information we also identified relative extrema and inflection points. To sketch a fairly accurate graph of $y = f(x)$ we also make use of

(v) The x-intercepts of $y = f(x)$,

(vi) the y-intercept of $y = f(x)$,

(vii) the horizontal asymptotes of $y = f(x)$, and

(viii) the vertical asymptotes of $y = f(x)$.

Example

The function $f(x) = \dfrac{(x-1)^2}{(x-3)^2}$ has derivatives

$$f'(x) = \frac{-4(x-1)}{(x-3)^3} \quad \text{and} \quad f''(x) = \frac{8x}{(x-3)^4}$$

Sketch the graph of $y = f(x)$ using the

(i) x-intercepts

(ii) y-intercepts

(iii) vertical asymptotes

(iv) horizontal asymptotes

(v) intervals of increase/decrease

(vi) local extreme values

(vii) intervals of concavity

(viii) inflection points

Example 2

Suppose we have analyzed the function $y = f(x)$ and found the following information:

(i) The domain of f is $(-\infty, 1) \cup (1, \infty)$.

(ii) $f(x)$ has the following function values:

x	-3	-2	-1	$-1/2$	0	$1/2$	3	4
$f(x)$	$3/2$	2	1	0	$-1/2$	0	-1	$-3/2$

(iii) $\lim_{x\to-\infty} f(x) = 1$, $\lim_{x\to\infty} f(x) = -2$

(iv) $\lim_{x\to 1^-} f(x) = \infty$, $\lim_{x\to 1^+} f(x) = -\infty$

(v) $f'(-2) = f'(0) = f'(3) = 0$

(vi) $f'(x) > 0$ on $(-\infty, -2)$, $(0, 1)$ and $(1, 3)$;
$f'(x) < 0$ on $(-2, 0)$ and $(3, \infty)$

(vii) $f''(-3) = f''(-1) = f''(4) = 0$

(viii) $f''(x) > 0$ on $(-\infty, -3)$, $(-1, 1)$ and $(4, \infty)$;
$f''(x) < 0$ on $(-3, -1)$ and $(1, 4)$

Sketch the graph of $y = f(x)$.