

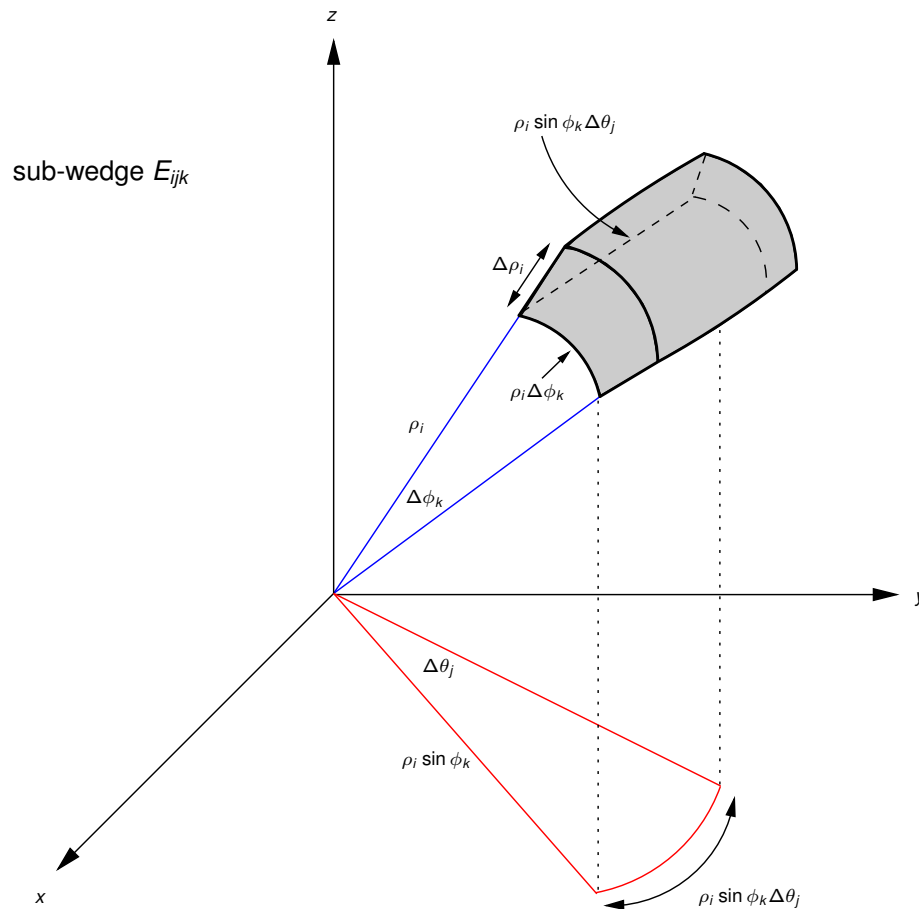
Spherical Coordinates

Consider the spherical wedge

$$E = \{(\rho, \theta, \phi)_{\text{spherical}} : a \leq \rho \leq b, \alpha \leq \theta \leq \beta, c \leq \phi \leq d\}$$

where $\beta - \alpha \leq 2\pi$. We wish to compute $\iiint_E f(x, y, z) dV$.

Partition the ρ , θ and ϕ intervals into ℓ , m and n subintervals, respectively, which defines small sub-wedges:



Here E_{ijk} has approximate volume

$$V_{ijk} \approx (\Delta \rho_i)(\rho_i \sin \phi_k \Delta \theta_j)(\rho_i \Delta \phi_k) = \rho_i^2 \sin \phi_k \Delta \rho_i \Delta \theta_j \Delta \phi_k$$

Consequently,

$$\iiint_E f(x, y, z) dV = \int_{\phi=c}^d \int_{\theta=\alpha}^{\beta} \int_{\rho=a}^b f(\rho \sin \phi \cos \theta, \rho \sin \phi \sin \theta, \rho \cos \phi) \rho^2 \sin \phi d\rho d\theta d\phi$$