

1. Given nonzero vectors  $\mathbf{a}$  and  $\mathbf{b}$  in  $\mathbb{R}^3$ , show that  $\mathbf{v} = |\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$  bisects the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .
2. Show that the vectors  $|\mathbf{b}|\mathbf{a} + |\mathbf{a}|\mathbf{b}$  and  $|\mathbf{b}|\mathbf{a} - |\mathbf{a}|\mathbf{b}$  are orthogonal.
3. Find an equation of the plane that contains the point  $P(3, -1, 2)$  and the line  $\mathbf{r}(t) = \langle 2, -1, 0 \rangle + t\langle 2, 3, 0 \rangle$ .
4. Find a unit vector that is parallel to both the planes  $8x + y + z = 1$  and  $x - y - z = 0$ .
5. Find a unit vector that makes an angle of  $30^\circ$  with  $\hat{\mathbf{i}}$  and makes equal angles with  $\hat{\mathbf{j}}$  and  $\hat{\mathbf{k}}$ .
6. Let  $f(x, y) = x^2 e^{-xy}$ 
  - (a) Find a normal to the surface  $z = f(x, y)$  at the point  $(1, 2)$ .
  - (b) Find the equation of the tangent plane to the surface  $z = f(x, y)$  at the point  $(1, 2)$ .
  - (c) What point on the surface  $z = x^2 - y^2$  has a tangent plane parallel to the plane in (b)?
7. Find an equation of the tangent plane to  $(\cos x)(\cos y)e^z = 0$  at  $(\pi/2, 1, 0)$ .
8. At time  $t = 0$  a particle is ejected from the surface  $x^2 + 2y^2 + 3z^2 = 6$  at the point  $(1, 1, 1)$  in a direction normal to the surface at a speed of 10 units per second. At what time does the particle first reach the surface of the sphere  $x^2 + y^2 + z^2 = 103$ ?
9. Let  $f(x, y) = 5ye^x - e^{5x} - y^5$ . Show that  $f$  has a unique critical point corresponding to a local maximum, but that  $f$  has no absolute maximum.