Question 1: Determine if the space curve with parametric equations $x = t\cos(t)$, $y = t\sin(t)$, $z = t$ lies on the surface $z^2 = x^2 + y^2$.

[5]

Question 2: Determine the point where the tangent line to $\mathbf{r}(t) = \langle 1+2\sqrt{t},\, t^3 - t,\, t^3 + t\rangle$ at $(3, 0, 2)$ intersects the xy-plane.

[5]

Question 3: A projectile of mass 2 kg is launched from the origin in the direction of the positive x-axis with an initial speed of 40 m/s at an angle of 60° to the ground. The wind applies a constant horizontal opposing force of 1 N. The resulting equation of motion governing the projectile's flight is

$$2\mathbf{a}(t) = -\mathbf{i} - 2g\mathbf{j}$$

where $g = 9.8$ m/s^2 is acceleration due to gravity. How far from the launch site does the projectile land?

[10]

Question 4: Neatly draw a contour map for $f(x, y) = ye^x$ showing contours corresponding to $k = -1, 0, 1$. Label the three contours.

[5]

Question 5: The following limit exists; find it: $\lim_{(x,y)\to(1,1)} \dfrac{x^2 - y^2}{x^2y - x^3}$

[5]

Question 6: Show that the following limit does not exist (a bit trickier): $\lim_{(x,y)\to(0,0)} \dfrac{x^2 y e^y}{x^4 + 4y^2}$

[5]

Question 7: Let $g(x, y) = \dfrac{x + 2y}{x^2 + y^2}$. Compute $g_x(1, 2) - g_y(1, 2)$.

[5]

Question 8: Let $f(x, y, z) = x \sin(y^2 - z^2)$. Compute $\dfrac{\partial^2 f}{\partial x^2} - \dfrac{\partial^2 f}{\partial z^2}$.

[5]

Question 9: Let $f(x, y, z) = xy^2z^3 + \arcsin(x\sqrt{z})$. Find f_{xzy}. (You may assume that Clairaut's Theorem applies.)

[5]