Question 1: The following function has domain $[0, \infty)$:

$$f(x) = \begin{cases} \dfrac{\sqrt{x}-2}{x-4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases}$$

Determine the value of c so that f is continuous at $x = 4$.

[5]

Question 2: Use the Intermediate Value Theorem to show that the equation $\sin(x) = 3 - 2x$ has a solution on the interval $[0, 2\pi]$.

[5]

Question 3: Evaluate the following limits, if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate limits.)

(a) $\displaystyle\lim_{x\to-\infty} \frac{6x^5 - 7x^3 - 5}{7x^6 - 6x^5 + 5}$

[3]

(b) $\displaystyle\lim_{x\to2^+} \frac{\cos(\pi x)}{2 - x}$

[3]

(c) $\displaystyle\lim_{x\to1} \frac{2 - x}{(x-1)^3}$

[4]

Question 4:

(a) Use <u>the limit definition of the derivative</u> to find $f'(x)$ if $f(x) = \dfrac{5}{x+7}$. Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

[8]

(b) Now check your work in part (a) by finding $\dfrac{d}{dx}\left[\dfrac{5}{x+7}\right]$ using derivative rules.

[2]

Question 5: Find the following derivatives (it is not necessary to simplify your answers, but marks will be deducted for improper use of notation):

(a) $f(x) = 3\sqrt{x} - \dfrac{2}{3x} + \pi^2$

[2]

(b) $y = (t^3 - 3\cos(t))(2t - \sqrt{\pi})$

[3]

(c) $g(x) = \dfrac{x^2 - \sqrt[3]{x}}{\tan(x)}$

[3]

(d) $y = \dfrac{\sin(\theta)}{2} - \dfrac{2}{\sin(\theta)}$

[2]

Question 6: The position of a particle along a straight line at time t is given by

$$s(t) = \frac{t^3}{3} + qt(t+1)$$

where q is a constant. At time $t = 1$ the velocity and acceleration are the same. Determine the value of the constant q.

[5]

Question 7: Find an equation of the tangent line to $y = \dfrac{\pi^2 \cos(x)}{x}$ at the point where $x = \pi$.

[5]