

Curve Sketching

So far we have seen that

- (i) If $f'(x) > 0$ on an interval then the graph of $y = f(x)$ is increasing on the interval;
- (ii) If $f'(x) < 0$ on an interval then the graph of $y = f(x)$ is decreasing on the interval;
- (iii) If $f''(x) > 0$ on an interval then the graph of $y = f(x)$ is concave up on the interval;
- (iv) If $f''(x) < 0$ on an interval then the graph of $y = f(x)$ is concave down on the interval.

Using this information we also identified relative extrema and inflection points. To sketch a fairly accurate graph of $y = f(x)$ we also make use of

- (v) The x -intercepts of $y = f(x)$,
- (vi) the y -intercept of $y = f(x)$,
- (vii) the horizontal asymptotes of $y = f(x)$, and
- (viii) the vertical asymptotes of $y = f(x)$.

Example

The function $f(x) = \frac{(x-1)^2}{(x-3)^2}$ has derivatives

$$f'(x) = \frac{-4(x-1)}{(x-3)^3} \quad \text{and} \quad f''(x) = \frac{8x}{(x-3)^4}$$

Sketch the graph of $y = f(x)$ using the

- (i) x -intercepts
- (ii) y -intercepts
- (iii) vertical asymptotes
- (iv) horizontal asymptotes
- (v) intervals of increase/decrease
- (vi) local extreme values
- (vii) intervals of concavity
- (viii) inflection points

Example 2

Suppose we have analyzed the function $y = f(x)$ and found the following information:

(i) The domain of f is $(-\infty, 1) \cup (1, \infty)$.

(ii) $f(x)$ has the following function values:

x	-3	-2	-1	-1/2	0	1/2	3	4
$f(x)$	3/2	2	1	0	-1/2	0	-1	-3/2

(iii) $\lim_{x \rightarrow -\infty} f(x) = 1$, $\lim_{x \rightarrow \infty} f(x) = -2$

(iv) $\lim_{x \rightarrow 1^-} f(x) = \infty$, $\lim_{x \rightarrow 1^+} f(x) = -\infty$

(v) $f'(-2) = f'(0) = f'(3) = 0$

(vi) $f'(x) > 0$ on $(-\infty, -2)$, $(0, 1)$ and $(1, 3)$;
 $f'(x) < 0$ on $(-2, 0)$ and $(3, \infty)$

(vii) $f''(-3) = f''(-1) = f''(4) = 0$

(viii) $f''(x) > 0$ on $(-\infty, -3)$, $(-1, 1)$ and $(4, \infty)$;
 $f''(x) < 0$ on $(-3, -1)$ and $(1, 4)$

Sketch the graph of $y = f(x)$.