Question 1 [12 points]: Evaluate the following limits, if they exist. If a limit does not exist because it is $\pm\infty$, state which it is and include an explanation of your reasoning. You may use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

(a) $\lim_{x\to -5} \frac{x^2+3x-10}{x^2+8x+15}$

(b) $\lim_{x\to 9} \frac{4-\sqrt{x+7}}{9-x}$

(c) $\lim_{x\to 0} \frac{\tan(2x)+3x}{\sin(2x)}$

(d) $\lim_{x\to 2^-} \frac{-4}{x^2-x-2}$

Question 2 [8 points]: Consider the function $f(x) = x^3 - 2x^2 - 1$.

(a) Find an equation of the tangent line to the graph of $f(x)$ at $x = -2$.

(b) Find the x-value(s) where the tangent line to the graph of $f(x)$ is parallel to the line $y = 4x - 2$.

Question 3 [15 points]: Differentiate the following functions (you do not have to simplify your answers):

(a) $y = 4\ln(3x^4 - 5x^2)$

(b) $y = \dfrac{e^x - x}{x^2 + \sin x}$

(c) $y = 5^x \sec x$

(d) $y = \left(5x^3 - \dfrac{7}{x^2}\right) e^{\cos x}$

(e) $y = \tan\left(\sqrt{x^3 - \log_5 x}\right)$

Question 4 [5 points]: Use the definition of the derivative to find $f'(x)$ for $f(x) = \dfrac{3}{x+7}$. (A score of 0 will be given if $f'(x)$ is found using the differentiation rules.)

Question 5 [10 points]:

(a) Use implicit differentiation to find $\dfrac{dy}{dx}$ for $e^{xy^2} = 4x^2 - \cos y$.

(b) Use logarithmic differentiation to find $\dfrac{dy}{dx}$ for $y = \dfrac{\sin^{10} x}{7^{(x^2-4x+1)}}$.

Question 6 [10 points]:

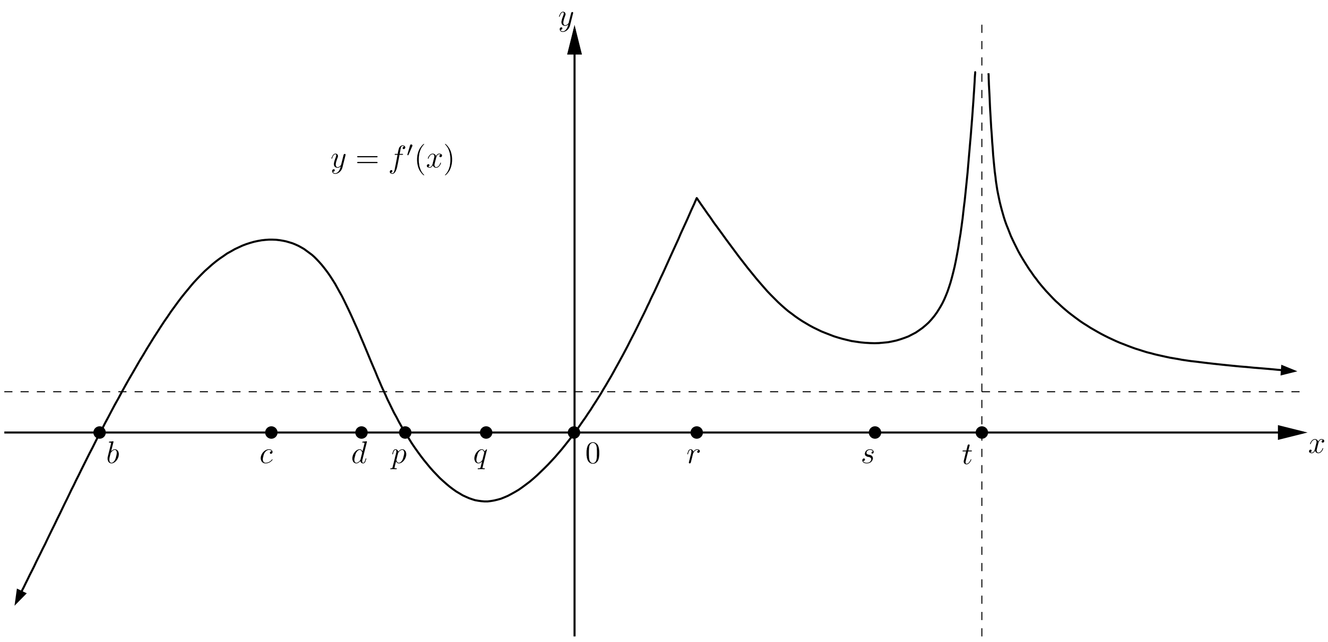
(a) Find the general antiderivative for each of the following functions:

(i) $f(x) = -\sec^2 x - 4\sin x + 7e^x$

(ii) $f(x) = \dfrac{3x^6 - 4x + 2\sqrt[4]{x^3}}{x^2}$

(b) Find the function $g(t)$ such that $g'(t) = 5t^4 + 9t^2$ and $g(-1) = 7$.

Question 7 [10 points]: The graph of $f'(x)$ is shown below (note this is the graph of $f'(x)$, not $f(x)$):



(a) On what interval(s) is f decreasing?

(b) At what x-value(s) does $f(x)$ have local minima?

(c) On what interval(s) is the graph of $f(x)$ concave up?

(d) At what x-value(s) does the graph of $f(x)$ have inflection points?

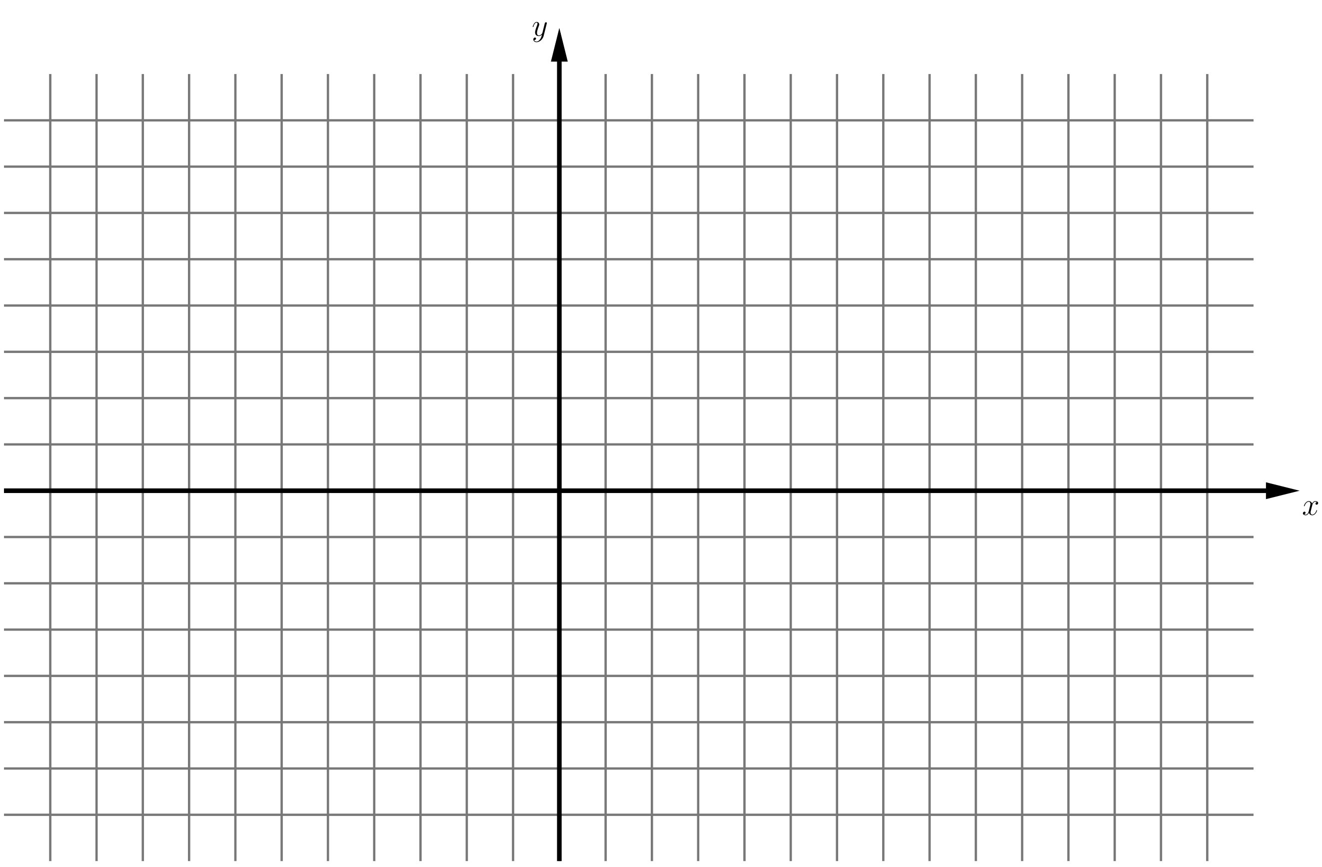
(e) At what x-value(s) does $f''(x)$ not exist?

Question 8 [5 points]: You have completed the analysis of a function $f(x)$ and found the information listed below. Sketch the graph of $y = f(x)$.

- The domain of $f(x)$ is $(-\infty, 2), (2, \infty)$.
- $f(x)$ has the following function values:

x	-6	-3	-1	0	3	5	9
$f(x)$	-2	0	3	5	0	2	-1

- $\lim_{x\to\infty} f(x) = -3$, $\lim_{x\to-\infty} f(x) = \infty$
- $\lim_{x\to 2} f(x) = -\infty$
- $f'(-6) = f'(0) = f'(5) = 0$
- $f'(x) > 0$ on $(-6, 0)$ and $(2, 5)$
- $f'(x) < 0$ on $(-\infty, -6)$, $(0, 2)$ and $(5, \infty)$
- $f''(-1) = f''(9) = 0$
- $f''(x) > 0$ on $(-\infty, -1)$ and $(9, \infty)$
- $f''(x) < 0$ on $(-1, 2)$ and $(2, 9)$



Question 9 [12 points]: The function $f(x) = \frac{x}{x^2+1}$ has $f'(x) = \frac{1-x^2}{(x^2+1)^2}$ and $f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$.

(a) Find the intervals on which $f(x)$ is increasing or decreasing.

(b) Find the local maximum and minimum values of $f(x)$.

(c) Find the intervals on which $f(x)$ is concave up or concave down.

(d) Find the inflection points of $f(x)$.

Question 10 [8 points]: A straight wire is 60 cm long. The wire is bent into the shape of an L, where the bend forms a right angle. What is the shortest possible distance between the two ends of the bent wire?

Question 11 [10 points]: Two ships, the Erebus and the Terror, are sailing the high seas in search of the Northwest Passage. At 6:00 am, the Erebus is 30 km north of the Terror. The Erebus is sailing west at 8 km/hr and the Terror is sailing east at 12 km/hr. How fast is the distance between the ships changing at 8:00 am?

Question 12 [10 points]: A cylindrical can with a bottom but no top is to be made from 300π square meters of aluminum. Find the largest possible volume of such a can. (Note that the volume of cylinder is $V = \pi r^2 h$.)

Question 13 [5 points]: Consider the function $f(x) = ax^3 + bx^2 + c$. Find the values of a, b and c so that $(-1, 0)$ is a point on $f(x)$ and so that $f(x)$ has a point of inflection at $(1, 1)$.