Question 1 [12 points]: Evaluate the following limits (you may use the fact that $\lim_{x\to 0}\frac{\sin x}{x} = 1$):

(a) $\displaystyle\lim_{x\to\infty}\frac{2e^x - 7e^{-x} + 3}{5e^x + 4e^{-x} - 1}$

(b) $\displaystyle\lim_{x\to 5}\frac{x^2 - 7x + 10}{x^2 - 8x + 15}$

(c) $\displaystyle\lim_{x\to 0}\frac{\tan(3x)\cos(2x)}{x}$

Question 2 [8 points]:

(a) Let $f(x) = \sqrt{x+1}$. Use the <u>definition of the derivative</u> to find $f'(x)$. (No credit will be given if $f'(x)$ is found using differentiation rules.)

(b) Find the linear approximation to $f(x) = \sqrt{x+1}$ at $a = 3$.

Question 3 [15 points]: Differentiate the following functions (you do not need to simplify your answers):

(a) $y = 5\cos\left(x^7 - 8x\right)$

(b) $y = e^x \cot x$

(c) $y = \dfrac{2\sqrt{x}}{e^x - \pi x}$ (Use the Quotient Rule.)

(d) $y = x^3 4^{\tan x}$

(e) $y = \sqrt[5]{\log_7\left(x^2 - \cos x\right)}$

Question 4 [10 points]:

(a) Find the general antiderivative of the following functions:

(i) $f(x) = \sec^2 x + 3\sin x - \dfrac{e^x}{2}$

(ii) $y = \dfrac{2x^4 - \sqrt[3]{x^2} + 7}{x}$

(b) Find the function $f(t)$ with $f'(t) = 4t^3 + 2t$ and $f(1) = 9$.

Question 5 [10 points]:

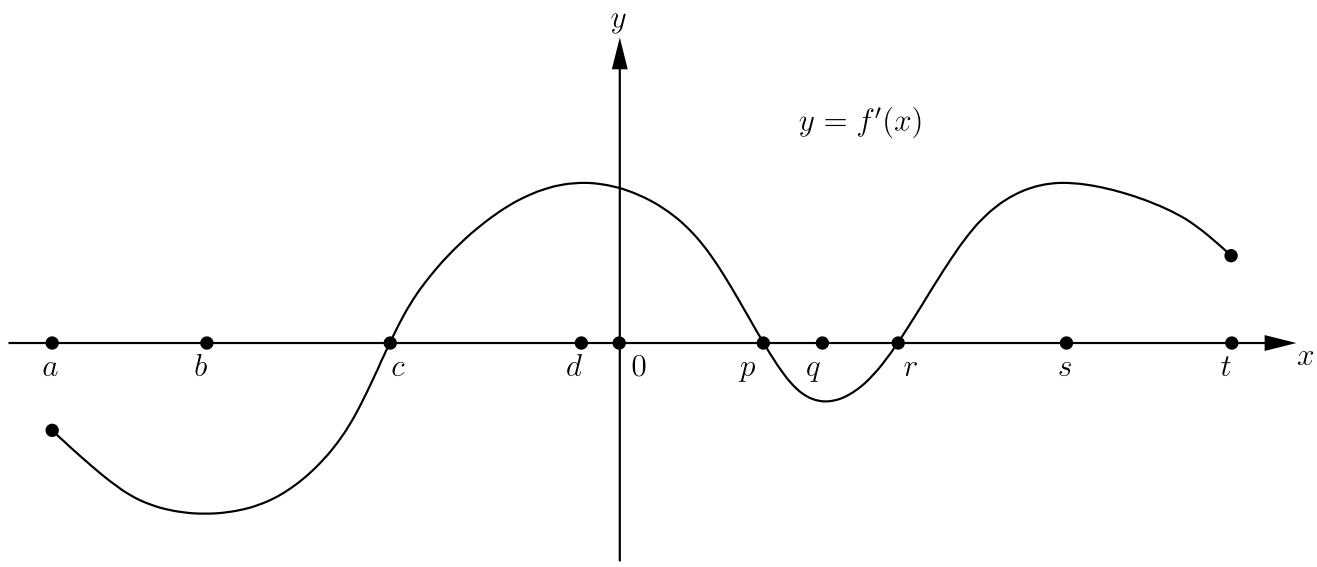
(a) Find $\dfrac{dy}{dx}$ by implicit differentiation:

$$xy + e^{3x} = \sin(x + y)$$

(b) Use logarithmic differentiation to find $\dfrac{dy}{dx}$:

$$y = \frac{3^{\sin x}}{(2x+1)^{10}}$$

Question 6 [8 points]: Consider the graph of $f'(x)$ below (note this is the graph of $f'(x)$, not $f(x)$):



(a) On what interval(s) is f increasing?

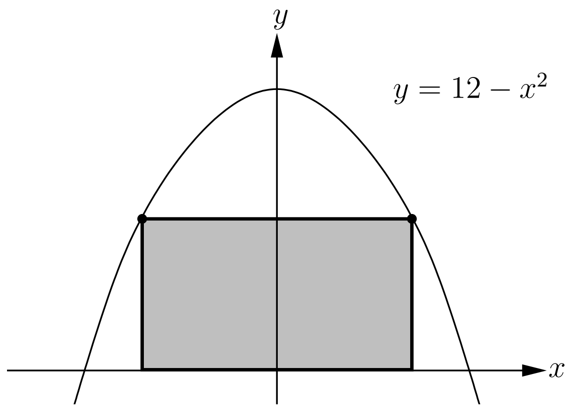
(b) At what value(s) of x does f have local minima?

(c) On what interval(s) is the graph of f concave down?

(d) At what value(s) of x does the graph of f have points of inflection?

Question 7 [10 points]: A water tank has the shape of an inverted circular cone with top radius 6 metres and height 18 metres. Water is pumped into the tank at a rate of 10 cubic metres per minute. How fast is the water level rising when the water is 2 metres deep? (Recall, the volume of of a right circular cone of height h and radius r is $V = \frac{1}{3}\pi r^2 h$.)

Question 8 [10 points]: A rectangle with base on the x-axis has its upper vertices on the curve $y = 12 - x^2$ (see figure below.) Find the maximum area of such a rectangle.



Question 9 [10 points]: A rectangular box without a top is constructed from cardboard. The length of the box is equal to three times the width. The volume of the box is 18 cubic metres. Find the <u>dimensions</u> of the box which minimize the amount of cardboard used.

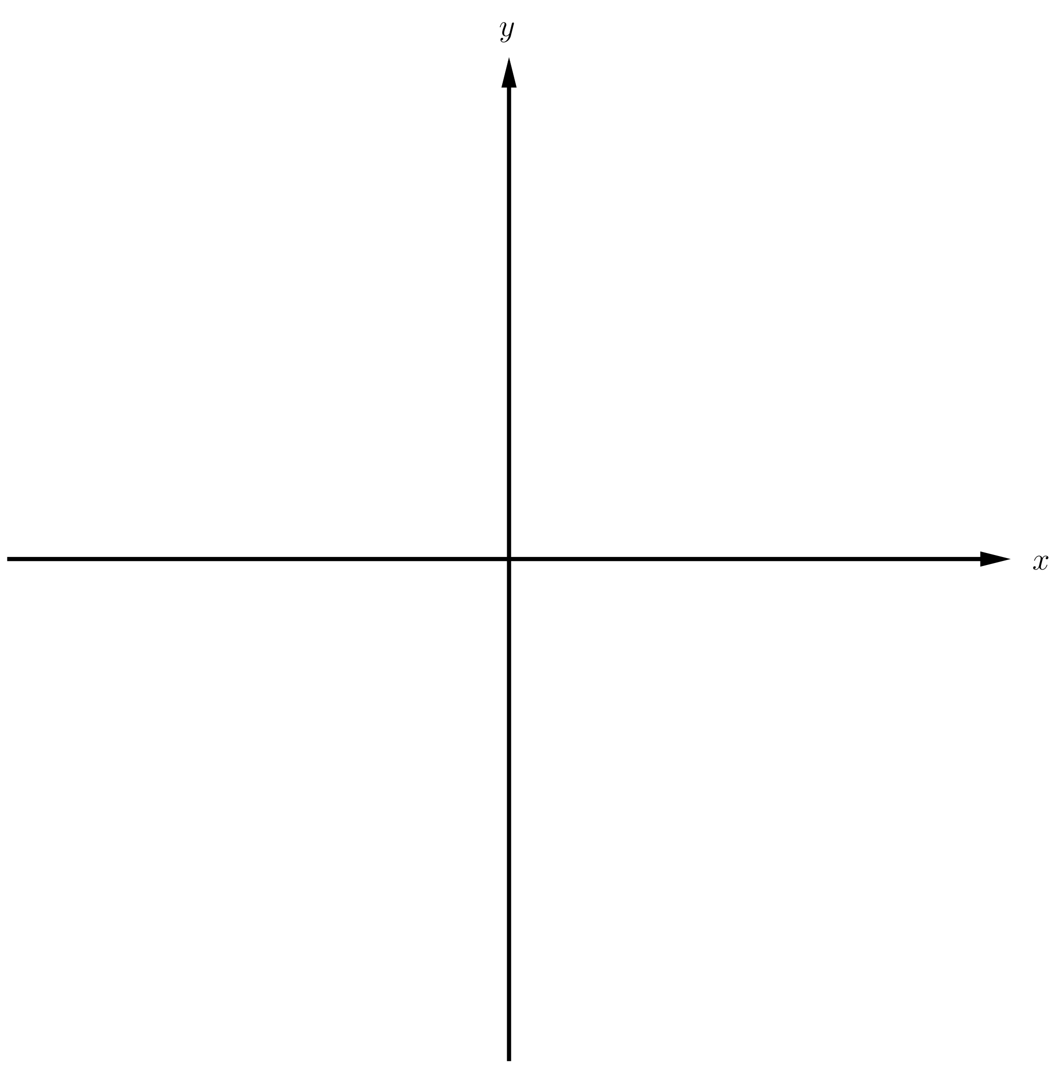
Question 10 [5 points]: Suppose you have analyzed a function and found the following:

1. the domain of f is $(-\infty, -4)$, $(-4, \infty)$
2. f has the following function values:

x	-6	-5	-1	0	2	3	5	8
$f(x)$	5	0	0	-1	-2	0	4	0

3. $\lim_{x\to-\infty} f(x) = 2$, $\quad \lim_{x\to\infty} f(x) = -\infty$
4. $\lim_{x\to-4^+} f(x) = \infty$, $\quad \lim_{x\to-4^-} f(x) = -\infty$
5. $f'(-6) = f'(2) = f'(5) = 0$
6. $f'(x) > 0$ on $(-\infty, -6)$ and $(2, 5)$
7. $f'(x) < 0$ on $(-6, -4)$, $(-4, 2)$ and $(5, \infty)$
8. $f''(-8) = f''(1) = f''(3) = 0$
9. $f''(x) > 0$ on $(-\infty, -8)$ and $(-4, 3)$
10. $f''(x) < 0$ on $(-8, -4)$ and $(3, \infty)$

Neatly sketch the graph of $y = f(x)$.



Question 11 [12 points]: The function $f(x) = x^4 e^{-x}$ has first derivative $f'(x) = (4x^3 - x^4)e^{-x}$ and second derivative $f''(x) = (x^4 - 8x^3 + 12x^2)e^{-x}$.

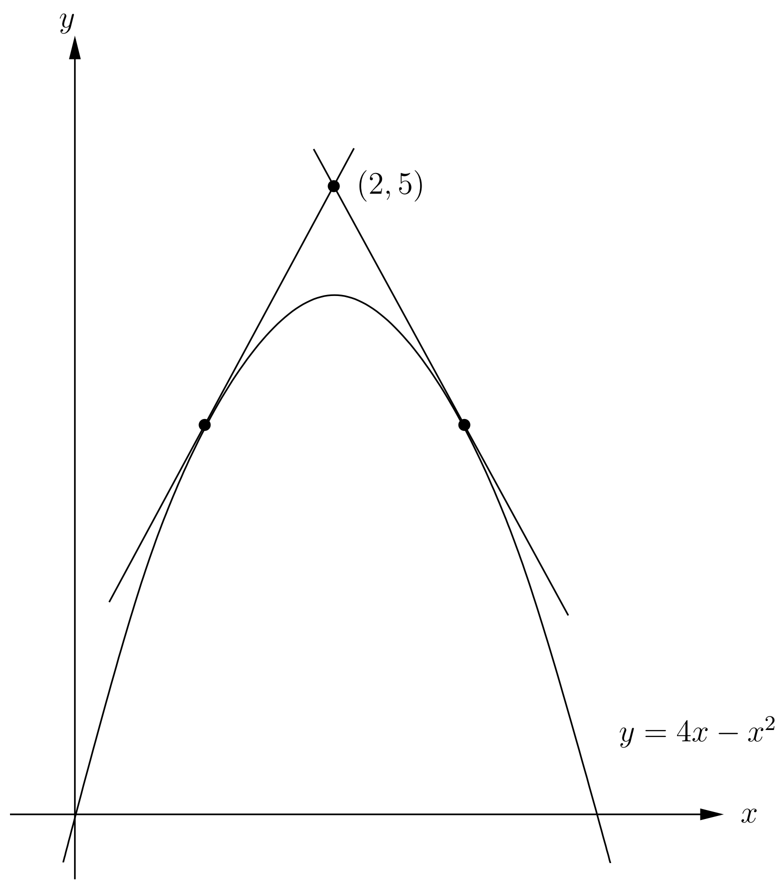
(a) Find the intervals on which $f(x)$ is increasing or decreasing.

(b) Find the local maximum and minimum values of $f(x)$.

(c) Find the intervals on which $f(x)$ is concave up or concave down.

(d) Find the inflection points of $f(x)$.

Question 12 [5 points]: There are two tangent lines to the curve $y = 4x - x^2$ that pass through the point $(2, 5)$; see the diagram below. Find the coordinates of the points where these tangent lines intersect the parabola.



Question 13 [5 points]: Suppose $f(x) = axe^{bx}$ where a and b are constants. If $f(1/3) = 1$ and $y = f(x)$ has a maximum at $x = 1/3$, find the values of a and b.