**Question 1:** The following function has domain $[0, \infty)$:

$$f(x) = \begin{cases} \dfrac{\sqrt{x}-2}{x-4} & \text{if } x \neq 4 \\ c & \text{if } x = 4 \end{cases}$$

Determine the value of $c$ so that $f$ is continuous at $x = 4$.

Need $\lim_{x \to 4} f(x) = f(4)$

$\Rightarrow \lim_{x \to 4} \dfrac{\sqrt{x}-2}{x-4} = c$

$\Rightarrow \lim_{x \to 4} \dfrac{(\sqrt{x}-2)(\sqrt{x}+2)}{(x-4)(\sqrt{x}+2)} = c$

$\Rightarrow \lim_{x \to 4} \dfrac{\cancel{(x-4)}\ 1}{\cancel{(x-4)}(\sqrt{x}+2)} = c$

$\boxed{\therefore\ c = \frac{1}{4}}$

**[5]**

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**Question 2:** Use the Intermediate Value Theorem to show that the equation $\sin(x) = 3 - 2x$ has a solution on the interval $[0, 2\pi]$.

$\sin(x) = 3 - 2x$

$\Rightarrow \sin(x) - 3 + 2x = 0$.

Let $f(x) = \sin(x) - 3 + 2x$.

$f$ is continuous on $[0, 2\pi]$,

$f(0) < 0$, $f(2\pi) > 0$, so

by the Intermediate Value Theorem

$f(c) = 0$ for some $0 < c < 2\pi$.

That is, $\sin(c) - 3 + 2c = 0$

So

$\sin(c) = 3 - 2c$

for some $c$ in $[0, 2\pi]$.

**[5]**

**Question 3:** Evaluate the following limits, if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate limits.)

**(a)** $\lim\limits_{x\to-\infty} \dfrac{6x^5 - 7x^3 - 5}{7x^6 - 6x^5 + 5} = \lim\limits_{x\to-\infty} \dfrac{\cancel{x^5}\left(6 - \frac{7}{x^2} - \frac{5}{x^5}\right)}{x^{\cancel{6}}\left(7 - \frac{6}{x} + \frac{5}{x^6}\right)}$

$= \boxed{0}$

[3]

**(b)** $\lim\limits_{x\to 2^+} \dfrac{\cos(\pi x)}{2 - x}$ $\begin{matrix} \} \to 1 \\ \} \to 0^- \end{matrix}$

$= \boxed{-\infty}$

[3]

**(c)** $\lim\limits_{x\to 1} \dfrac{2 - x}{(x-1)^3}$

$\lim\limits_{x\to 1^+} \dfrac{2-x}{(x-1)^3} \begin{matrix} \} \to 1 \\ \} \to 0^+ \end{matrix}$

$= +\infty$

$\lim\limits_{x\to 1^-} \dfrac{2-x}{(x-1)^3} \begin{matrix} \} \to 1 \\ \} \to 0^- \end{matrix}$

$= -\infty.$

not the same!

so $\lim\limits_{x\to 1} \dfrac{2-x}{(x-1)^3}$ does not exist

[4]

**Question 4:**

**(a)** Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{5}{x+7}$ . Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{5}{x+h+7} - \frac{5}{x+7}\right]$$

$$= \lim_{h\to 0} \frac{5}{h}\left[\frac{(x+7)-(x+h+7)}{(x+h+7)(x+7)}\right]$$

$$= \lim_{h\to 0} \frac{5}{\cancel{h}}\left[\frac{-\cancel{h}}{(x+h+7)(x+7)}\right]$$

$$= \boxed{\frac{-5}{(x+7)^2}}$$

**[8]**

**(b)** Now check your work in part (a) by finding $\dfrac{d}{dx}\left[\dfrac{5}{x+7}\right]$ using derivative rules.

$$\frac{d}{dx}\left[\frac{5}{x+7}\right] = 5\frac{d}{dx}\left[\frac{1}{x+7}\right]$$

$$= 5 \cdot \frac{-1}{(x+7)^2} \cdot 1 \quad \Big\} \text{ Reciprocal Rule.}$$

$$= \boxed{\frac{-5}{(x+7)^2}}$$

**[2]**

**Question 5:** Find the following derivatives (it is not necessary to simplify your answers, but marks will be deducted for improper use of notation):

**(a)** $f(x) = 3\sqrt{x} - \dfrac{2}{3x} + \pi^2$ $= 3x^{1/2} - \frac{2}{3}x^{-1} + \pi^2$

$\boxed{\therefore f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + \frac{2}{3}x^{-2}}$

**[2]**

**(b)** $y = (t^3 - 3\cos t)(2t - \sqrt{\pi})$

$\boxed{y' = (3t^2 + 3\sin(t))(2t - \sqrt{\pi}) + (t^3 - 3\cos(t))(2)}$

**[3]**

**(c)** $g(x) = \dfrac{x^2 - \sqrt[3]{x}}{\tan(x)}$ $= \dfrac{x^2 - x^{1/3}}{\tan(x)}$

$\boxed{g'(x) = \dfrac{\tan(x)\left(2x - \frac{1}{3}x^{-2/3}\right) - \left(x^2 - x^{1/3}\right)\sec^2(x)}{\tan^2(x)}}$

**[3]**

**(d)** $y = \dfrac{\sin(\theta)}{2} - \dfrac{2}{\sin(\theta)}$ $= \frac{1}{2}\sin(\theta) - 2\csc(\theta)$

$\boxed{y' = \frac{1}{2}\cos(\theta) + 2\csc(\theta)\cot(\theta)}$

**[2]**

**Question 6:** The position of a particle along a straight line at time $t$ is given by

$$s(t) = \frac{t^3}{3} + qt(t+1)$$

where $q$ is a constant. At time $t = 1$ the velocity and acceleration are the same. Determine the value of the constant $q$.

$s(t) = \left(\frac{1}{3}\right)t^3 + q t^2 + q t$

$s'(t) = t^2 + 2q t + q$

$s''(t) = 2t + 2q$

$s'(1) = s''(1)$, so $1^2 + \cancel{2q(1)} + q = 2(1) + \cancel{2q}$

$\boxed{\therefore\ q = 1}$

[5]

**Question 7:** Find an equation of the tangent line to $y = \dfrac{\pi^2 \cos(x)}{x}$ at the point where $x = \pi$.

At $x = \pi$, $y = \dfrac{\pi^2 \cos(\pi)}{\pi} = -\pi$, so $(\pi, -\pi)$ is a point on the tangent line.

Slope of tangent line is

$$y'\Big|_{x=\pi} = \pi^2\left(\frac{-x\sin(x) - \cos(x)\cdot 1}{x^2}\right)\Bigg|_{x=\pi}$$

$$= \cancel{\pi^2}\left(\frac{-\cancel{\pi\sin(\pi)}^{\,0} - \cos(\pi)}{\cancel{\pi^2}}\right)$$

$$= 1$$

$\therefore$ Equation is $y - (-\pi) = 1(x - \pi)$ or $y + \pi = x - \pi$ or $\boxed{y = x - 2\pi}$

[5]