Question 1: Write as a single simplified fraction: $\dfrac{x}{x^2-1} - \dfrac{5}{2x^2+x-3}$

$$= \frac{x}{(x-1)(x+1)} - \frac{5}{(x-1)(2x+3)}$$

$$= \frac{x(2x+3) - 5(x+1)}{(x-1)(x+1)(2x+3)}$$

$$= \boxed{\frac{2x^2-2x-5}{(x-1)(x+1)(2x+3)}}$$

[4]

Question 2: Solve for x: $\dfrac{x^3+3x^2-5x}{x^2+9} = 0$

$$\Rightarrow \quad x^3+3x^2-5x=0$$

$$x(x^2+3x-5)=0$$

$\therefore \quad x=0 \quad$ or $\quad x^2+3x-5=0$

$$x = \frac{-3 \pm \sqrt{3^2-4(1)(-5)}}{2}$$

$$= \frac{-3 \pm \sqrt{29}}{2}$$

$$\boxed{\therefore \; x = 0, \frac{-3+\sqrt{29}}{2}, \frac{-3-\sqrt{29}}{2}}$$

[3]

Question 3: Simplify: $\left(\dfrac{x^3}{\sqrt{xy}}\right)\left(\dfrac{2y^{2/3}}{(2xy)^3}\right)$

$$= \frac{\cancel{x^3}}{x^{1/2}y^{1/2}} \cdot \frac{\cancel{2}\,y^{2/3}}{2^{\cancel{3}\,2}\,\cancel{x^3}\,y^3}$$

$$= \frac{1}{4x^{1/2}y^{3+\frac{1}{2}-\frac{2}{3}}}$$

$$= \boxed{\frac{1}{4x^{\frac{1}{2}}y^{17/6}}} \quad \text{or} \quad \boxed{\frac{1}{4\sqrt{x}\sqrt[6]{y^{17}}}}$$

[3]

Question 4: Expand and simplify: $(t-5)^2 - 2(t+3)(8t-1)$

$$= t^2 - 10t + 25 - 16t^2 - 46t + 6$$

$$\boxed{= -15t^2 - 56t + 31}$$

[3]

Question 5: Rationalize and simplify: $\dfrac{\sqrt{2+h}+\sqrt{2-h}}{h} \cdot \dfrac{\sqrt{2+h}-\sqrt{2-h}}{\sqrt{2+h}-\sqrt{2-h}}$

$$= \frac{2+h-(2-h)}{h\left(\sqrt{2+h}-\sqrt{2-h}\right)}$$

$$= \frac{2h}{h\left(\sqrt{2+h}-\sqrt{2-h}\right)}$$

$$= \boxed{\frac{2}{\sqrt{2+h}-\sqrt{2-h}}}$$

[4]

Question 6: A certain line L has twice the slope of the line $2x+3y=7$ and the two lines intersect at $x=2$. Determine an equation for the line L. (For your final answer use any form of the equation of a line you wish.)

$2x+3y=7 \Rightarrow y = -\frac{2}{3}x + \frac{7}{3}$: slope $-\frac{2}{3}$, and at $x=2$, $y=1$.

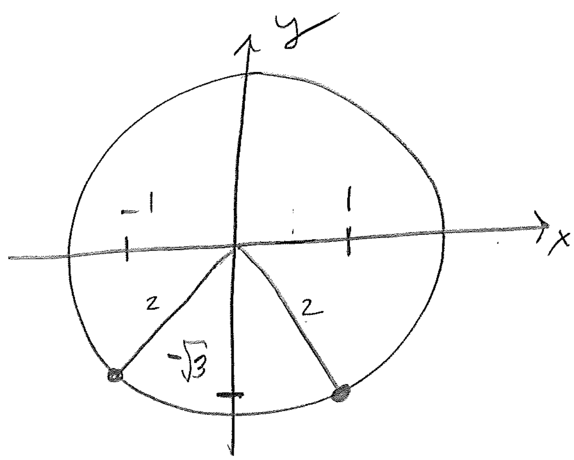
So L has slope $m = 2\left(-\frac{2}{3}\right) = -\frac{4}{3}$ and $(2,1)$ is a point on L.

So L has equation

$$\boxed{y-1 = -\frac{4}{3}(x-2)}$$

[3]

Question 7: Determine $\sin(5\pi/3) - \sec(4\pi/3)$. Express your answer as a single simplified fraction.



$$\sin\left(\frac{5\pi}{3}\right) - \sec\left(\frac{4\pi}{3}\right)$$

$$= \frac{-\sqrt{3}}{2} - \frac{2}{(-1)}$$

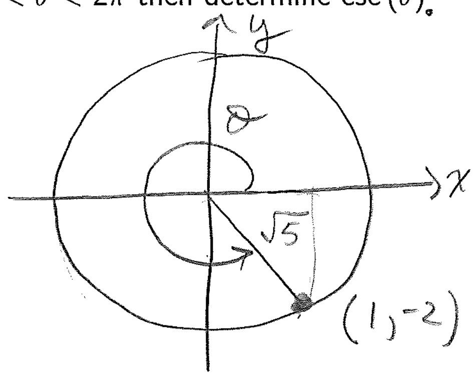
$$= \frac{-\sqrt{3}+4}{2}$$

or $\boxed{\frac{4-\sqrt{3}}{2}}$

[3]

Question 8: If $\tan(\theta) = -2$ where $3\pi/2 < \theta < 2\pi$ then determine $\csc(\theta)$.

$\tan(\theta) = \frac{-2}{1} = \frac{y}{x}$ ∴



∴ $r = \sqrt{1^2+2^2} = \sqrt{5}$

∴ $\csc(\theta) = \frac{\sqrt{5}}{-2} = \boxed{\frac{-\sqrt{5}}{2}}$

[3]

Question 9: Find all values of x in the interval $[0, 2\pi]$ for which $2\tan(x) = \sin(x)$.

$2\tan(x) = \sin(x)$

$2\,\frac{\sin(x)}{\cos(x)} - \sin(x) = 0$

$\sin(x)\left(\frac{2}{\cos(x)} - 1\right) = 0$

∴ $\sin(x) = 0$ or $\underbrace{\cos(x) = 2}_{\text{no solutions.}}$

$\boxed{∴ x = 0, \pi, 2\pi}$

[4]

Question 10: Determine the domain of $f(x) = \dfrac{x}{3-\sqrt{x-2}}$.

Require $x-2 \geq 0$ and $3-\sqrt{x-2} \neq 0$

so $x \geq 2$ and $x \neq 11$

so domain is $\boxed{[2,11) \cup (11,\infty).}$

[3]

Question 11: Evaluate and simplify the difference quotient $\dfrac{f(a+h)-f(a)}{h}$ where $f(x) = \dfrac{1}{x}$.

$$\frac{f(a+h)-f(a)}{h} = \frac{\left(\frac{1}{a+h} - \frac{1}{a}\right)}{h}$$

$$= \frac{1}{h}\left[\frac{a-(a+h)}{(a+h)a}\right]$$

$$= \frac{-\cancel{h}}{\cancel{h}(a+h)a}$$

$$= \boxed{\frac{-1}{(a+h)a}}$$

[4]

Question 12: A sphere (or ball) of radius r has volume $V = 4\pi r^3/3$ and surface area $S = 4\pi r^2$. Express the surface area as a function of the volume.

Want $S = f(V)$.

Since $V = \frac{4\pi r^3}{3}$, $r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}$.

$\therefore S = 4\pi r^2 = 4\pi\left[\left(\frac{3V}{4\pi}\right)^{\frac{1}{3}}\right]^2$

$= \frac{4\pi}{(4\pi)^{2/3}}(3V)^{2/3} = \boxed{(4\pi)^{1/3}(3V)^{2/3}}$

[3]

Question 13: Evaluate the following limits, if they exist:

(a) $\lim_{x\to -1} \frac{x-1}{x\sqrt{x^2+8}} = \frac{-2}{-1\sqrt{(-1)^2+8}} = \boxed{\frac{2}{3}}$

[2]

(b) $\lim_{x\to 2} \frac{x^2-4}{x^2+4x-12} \begin{matrix}\to \\ \to\end{matrix} \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+6)}$

$= \boxed{\frac{1}{2}}$

[2]

(c) $\lim_{h\to 0} \frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h} \begin{matrix}\to \\ \to\end{matrix} \text{"}\frac{0}{0}\text{"}$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{9-(3+h)^2}{9(3+h)^2}\right]$

$= \lim_{h\to 0} \frac{1}{h}\left[\frac{\cancel{9}-\cancel{9}-6h-h^2}{9(3+h)^2}\right]$

$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}(6+h)}{9(3+h)^2}\right]$

$= \frac{-6}{(9)(9)}$

$= \boxed{\frac{-2}{27}}$

[3]

(d) $\lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{4-x}} \begin{matrix}\to \\ \to\end{matrix} \text{"}\frac{0}{0}\text{"}$

$= \lim_{x\to 2} \frac{x-2}{\sqrt{x}-\sqrt{4-x}} \cdot \frac{\sqrt{x}+\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}}$

$= \lim_{x\to 2} \frac{(x-2)(\sqrt{x}+\sqrt{4-x})}{x-(4-x)}$

$= \lim_{x\to 2} \frac{\cancel{(x-2)}(\sqrt{x}+\sqrt{4-x})}{2\cancel{(x-2)}}$

$= \frac{\sqrt{2}+\sqrt{4-2}}{2}$

$= \frac{\sqrt{2}+\sqrt{2}}{2}$

$= \boxed{\sqrt{2}}$

[3]