Question 1: A couple borrows $400,000 to purchase a new house and plan to pay off the mortgage over 25 years. Payments are made at the end of each month and the mortgage rate is 4.5% compounded monthly. What are the required monthly payments? Round your final answer to the nearest dollar.

$V = 400{,}000$

$i = \frac{0.045}{12}$

$m = (25)(12)$

$$V = P\left[\frac{1-(1+i)^{-m}}{i}\right]$$

$$P = \frac{iV}{1-(1+i)^{-m}}$$

$$= \frac{\left(\frac{0.045}{12}\right)(400{,}000)}{1-\left(1+\frac{0.045}{12}\right)^{-(25)(12)}}$$

$$\doteq \boxed{\$2{,}223}$$

[5]

Question 2: The "Daily Grand" lottery game offers the winner of the top prize a choice:

(i) $1,000 each day for the rest of their life, or

(ii) a one time lump sum payment of $7,000,000.

If a winner has an estimated 40 more years to live and can access an investment paying 5% per year compounded daily, which is the better choice?

Convert the 7,000,000 into an annuity:

$V = 7{,}000{,}000$

$i = \frac{0.05}{365}$

$m = (40)(365)$

As above,

$$P = \frac{iV}{1-(1+i)^{-m}}$$

$$= \frac{\left(\frac{0.05}{365}\right)(7{,}000{,}000)}{1-\left(1+\frac{0.05}{365}\right)^{-(40)(365)}}$$

$$\doteq \$1109.$$

Since the $7,000,000 lump sum would provide a greater daily payout, taking the lump sum is better.

[5]

Question 3: For this question use the following sets:

$$U = \{a, b, c, d, e, f\}, \quad A = \{b, c\}, \quad B = \{c, d, e\}, \quad C = \{a, f\}$$

Determine the following:

(a) $\overline{A} \cap B$ $= \overline{\{b, c\}} \cap \{c, d, e\}$

$= \{a, d, e, f\} \cap \{c, d, e\}$

$= \boxed{\{d, e\}}$

[2]

(b) $(B \cap C) \cup (B \cap \overline{C})$ $= (\{c, d, e\} \cap \{a, f\}) \cup (\{c, d, e\} \cap \overline{\{a, f\}})$

$= \emptyset \cup (\{c, d, e\} \cap \{b, c, d, e\})$

$= \boxed{\{c, d, e\}}$

[2]

(c) $\overline{(\overline{A} \cap \overline{C})}$ $= \overline{(\overline{A})} \cup \overline{(\overline{C})}$

$= A \cup C$

$= \boxed{\{a, b, c, f\}}$

[3]

(d) $\overline{(A \cup C)}$ $= \overline{\{a, b, c, f\}}$ using ↖

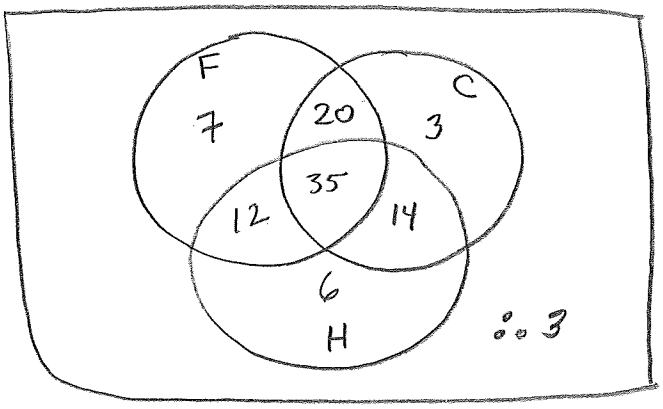
$= \boxed{\{d, e\}}$

[3]

Question 4: A doctor recorded symptoms reported by 100 patients who sought treatment for the flu. The results are:

✓74 reported fever;
✓ 72 reported chills;
✓ 67 reported headache;
✓55 reported both fever and chills;
✓47 reported both fever and headache;
✓49 reported both chills and headache;
✓35 reported all three symptoms (headache, fever and chills)

Let F: fever
C: chills
H: headache



(a) How many patients reported having fever but none of the other symptoms? 7

[3]

(b) How many of the patients reported having none of the symptoms? 3

[3]

Question 5: If sets A and B have 32 elements when combined, share 6 elements in common, and are both of the same size, determine the number of elements in each.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$32 = 2n(A) - 6$$

$$\therefore n(A) = \frac{32+6}{2} = 19$$

∴ Set A and B each have 19 elements.

[4]

Question 6:

(a) How many different 5-letter arrangements can be made use the letters from the word "PROBLEM"?

choices: $\underline{7}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}$

$\therefore$ number of arrangements is

$(7)(6)(5)(4)(3) = \boxed{2520}$

[3]

(b) How many of the arrangements in part (a) contain at least one of the letters "P" and "M"?

Let n = number of arrangement containing at least one of P & M

m = number of arrangents containing neither P nor M

$= (5)(4)(3)(2)(1)$

$= 120$

$\therefore n = 2520 - 120 = \boxed{2400}$

[3]

Question 7: What is the probability that a seven-digit phone number has one or more repeated digits?

S : all seven-digit phone numbers with non zero first digit

Choices: $\underline{9}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ $\}\ n(S) = 9(10^6) = 9000000$

E : phone number has one or more repeated digits

$\bar{E}$: phone number has no repeated digits

Choices: $\underline{9}\ \underline{9}\ \underline{8}\ \underline{7}\ \underline{6}\ \underline{5}\ \underline{4}$ $\}\ n(\bar{E}) = 544{,}320$

$\therefore P(E) = 1 - P(\bar{E}) = 1 - \frac{n(\bar{E})}{n(S)} = 1 - \frac{544{,}320}{9{,}000{,}000} \doteq \boxed{0.93952}$

[4]

Question 8: If 4 students are asked to pick a number between 1 and 10, what is the probability that at least two will choose the same number?

E: at least two choose the same number

$\overline{E}$: all choose different numbers.

S: all possible number choices

For $n(S)$: choices: 10 10 10 10 } so $n(S) = 10^4$

$n(\overline{E})$: 10 9 8 7 } so $n(\overline{E}) = (10)(9)(8)(7)$.

$\therefore P(E) = 1 - P(\overline{E}) = 1 - \frac{n(\overline{E})}{n(S)} = 1 - \frac{(10)(9)(8)(7)}{10^4} = \boxed{0.496}$

[4]

Question 9:

(a) A game consists of a single roll of a die. You receive \$1 if you roll a ⚀, \$3 if you roll a ⚂, and \$6 if you roll a ⚅. You receive \$0 for any other outcome. If you pay nothing to play, what is the expected value of your payout in such a game?

$$E = (1)P(⚀) + (3)P(⚂) + 6P(⚅)$$

$$= (1)\left(\tfrac{1}{6}\right) + (3)\left(\tfrac{1}{6}\right) + (6)\left(\tfrac{1}{6}\right)$$

$$= \frac{10}{6}$$

$$= \boxed{\$\tfrac{5}{3}} \text{ or } \boxed{\$1.67}$$

[3]

(b) Suppose now that you have to pay \$2 to play the game described in part (a). With this change, the prize payout for rolling a ⚅ will be modified to make the game fair. What should the new prize payout amount be for a ⚅ to ensure a fair game?

Let x be the new prize payout for a ⚅

We require

$$E = 0 = (-2)P(\{⚀, ⚃, ⚄\}) + (1-2)P(⚁) + (3-2)P(⚂) + (x-2)P(⚅)$$

So $(-2)\left(\frac{3}{6}\right) + (-1)\left(\frac{1}{6}\right) + (1)\left(\frac{1}{6}\right) + (x-2)\left(\frac{1}{6}\right) = 0$

$$-\frac{6}{6} + \frac{x}{6} - \frac{2}{6} = 0$$

$$\frac{x-8}{6} = 0$$

so $x = 8$

$\therefore$ new payout for a ⚅ should be \$8.

[3]