

Question 1: A couple borrows \$400,000 to purchase a new house and plan to pay off the mortgage over 25 years. Payments are made at the end of each month and the mortgage rate is 4.5% compounded monthly. What are the required monthly payments? Round your final answer to the nearest dollar.

[5]

Question 2: The “Daily Grand” lottery game offers the winner of the top prize a choice:

- (i) \$1,000 each day for the rest of their life, or
- (ii) a one time lump sum payment of \$7,000,000.

If a winner has an estimated 40 more years to live and can access an investment paying 5% per year compounded daily, which is the better choice?

[5]

Question 3: For this question use the following sets:

$$U = \{a, b, c, d, e, f\}, \quad A = \{b, c\}, \quad B = \{c, d, e\}, \quad C = \{a, f\}$$

Determine the following:

(a) $\overline{A} \cap B$

[2]

(b) $(B \cap C) \cup (B \cap \overline{C})$

[2]

(c) $\overline{(\overline{A} \cap \overline{C})}$

[3]

(d) $\overline{(A \cup C)}$

[3]

Question 4: A doctor recorded symptoms reported by 100 patients who sought treatment for the flu. The results are:

- 74 reported fever;
- 72 reported chills;
- 67 reported headache;
- 55 reported both fever and chills;
- 47 reported both fever and headache;
- 49 reported both chills and headache;
- 35 reported all three symptoms (headache, fever and chills)

(a) How many patients reported having fever but none of the other symptoms?

[3]

(b) How many of the patients reported having none of the symptoms?

[3]

Question 5: If sets A and B have 32 elements when combined, share 6 elements in common, and are both of the same size, determine the number of elements in each.

[4]

Question 6:

- (a) How many different 5-letter arrangements can be made use the letters from the word “PROBLEM”? (repetition of letters within an arrangement is not allowed)

[3]

- (b) How many of the arrangements in part (a) contain at least one of the letters “P” and “M”?

[3]

Question 7: A seven-digit phone number consists of digits chosen from $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ but the first digit cannot be zero. What is the probability that a seven-digit phone number has one or more repeated digits?

[4]

Question 8: If 4 students are asked to pick a number between 1 and 10, what is the probability that at least two will choose the same number?

[4]

Question 9:

(a) A game consists of a single roll of a die. You receive \$1 if you roll a 1, \$3 if you roll a 2, and \$6 if you roll a 3. You receive \$0 for any other outcome. If you pay nothing to play, what is the expected value of your payout in such a game?

[3]

(b) Suppose now that you have to pay \$2 to play the game described in part (a). With this change, the prize payout for rolling a 3 will be modified to make the game fair. What should the new prize payout amount be for a 3 to ensure a fair game?

[3]