Question 1: A hobby farmer raises goats and pigs. He wants to raise no more than 16 animals in total, of which at most 10 can be goats. It costs \$25 to raise a goat and \$75 to raise a pig, and the farmer has \$900 available for the project. Each goat produces \$12 in profit and each pig \$40 in profit. How many of each animal should the farmer raise to maximize total profit?

Graph paper is provided on the next page. Carefully set up the problem, identify your variables, neatly sketch equired graphs and state a clear conclusion.

Let x = number of goats.
y = number of pigs.

Maximize profit $P = 12x + 40y$

Subject to
$$\left.\begin{array}{r} x+y \leq 16 \\ x \leq 10 \\ 25x+75y \leq 900 \end{array}\right\} \text{ so } \begin{array}{r} x+y \leq 16 \\ x \leq 10 \\ x+3y \leq 36 \end{array}$$
$$x \geq 0$$
$$y \geq 0$$

Inequality	Line	Test Pt	Test Result
$x+y \leq 16$	$x+y=16$	$(0,0)$	$0+0 \leq 16$: T
$x \leq 10$	$x=10$	$(0,0)$	$0 \leq 10$: T
$x+3y \leq 36$	$x+3y=36$	$(0,0)$	$0+3(0) \leq 36$: T

Corner Points:

- By inspection; $(0,0), (0,12), (10,0)$
- Solving:

① $x+3y=36$
② $x+y=16$
①-②: $2y=20$
$y=10$
$\therefore x=6$
$\therefore (6,10)$

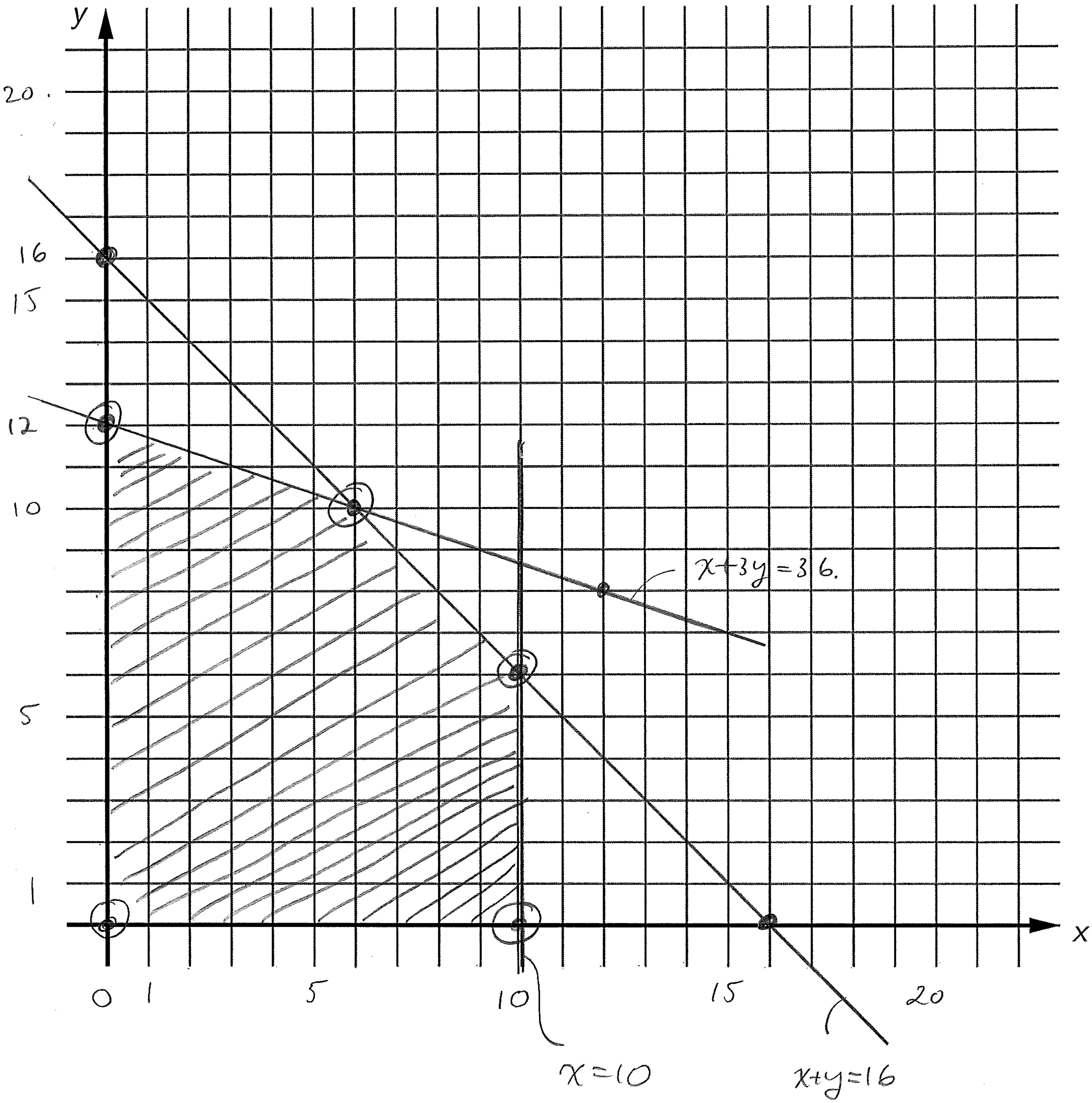
① $x+y=16$
② $x=10$
$\therefore y=6$
$\therefore (10,6)$

Corner Pt	$P=12x+40y$
$(0,0)$	0
$(0,12)$	480 ← max.
$(10,0)$	120
$(6,10)$	472
$(10,6)$	360

$\therefore$ Farmer should raise 0 goats and 12 pigs to maximize profit

[10]

Graph for **Question 1:**



Question 2: What amount must be invested at 7.5% compounded quarterly to have \$3000 in three years time? (Round your final answer to 2 decimal places.)

$r = 7.5\% = 0.075$

$n = 4$

$A = 3000$

$t = 3$

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$\Rightarrow P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

$P = \dfrac{3000}{\left(1+\frac{0.075}{4}\right)^{(4)(3)}}$

$\boxed{P = \$2400.54}$

[3]

Question 3: A person gets a payday loan of \$1500 which must be repaid in two weeks along with a fee of \$100. What rate of simple interest is being charged? (Round your final answer to 2 decimal places.)

$P = 1500$

$A = 1500 + 100 = 1600$

$t = \frac{2}{52} = \frac{1}{26}$

$A = P(1 + rt)$

$r = \left(\frac{A}{P} - 1\right)\left(\frac{1}{t}\right)$

$r = \left(\frac{1600}{1500} - 1\right)\left(\frac{1}{\frac{1}{26}}\right)$

$r = \left(\frac{1}{15}\right)(26)$

$r = 1.7\overline{3} \doteq \boxed{173.33\%}$

[3]

Question 4: What rate of interest compounded monthly is equivalent to an effective rate of 5%? (Round your final answer to 2 decimal places.)

$\cancel{P}\left(1+\frac{r}{12}\right)^{12} = \cancel{P}(1+R)$ where $R = 0.05$

$1 + \frac{r}{12} = (1+0.05)^{\frac{1}{12}}$

$r = 12\left[(1+0.05)^{\frac{1}{12}} - 1\right]$

$r \doteq 0.0489 = \boxed{4.89\%}$

[4]

Question 5: What rate of interest compounded quarterly will result in an investment tripling in 12 years? (Round your final answer to 2 decimal places.)

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 3P$

$n = 4$

$t = 12$

so $3\cancel{P} = \cancel{P}\left(1+\frac{r}{4}\right)^{(4)(12)}$

$r = \left(3^{1/48} - 1\right)(4)$

$r \doteq 0.0926 = \boxed{9.26\%}$

[3]

Question 6: Which investment is better: one that pays 7.5% interest compounded monthly or one that pays 7% compounded continuously?

Effective rate equivalent to 7.5% compounded monthly:

$R_1 = \left(1+\frac{0.075}{12}\right)^{12} - 1 \doteq 0.0776 = 7.76\%.$

Effective rate equivalent to 7% compounded continuously:

$R_2 = e^{0.07} - 1 \doteq 0.0725 = 7.25\%.$

Since $R_1 > R_2$, $\boxed{\text{The first investment is better.}}$

[4]

Question 7: How long does it take \$100 invested at 8% compounded semiannually to increase to \$300? (Round your answer to the nearest year.)

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 300$

$P = 100$

$r = 0.08$

$n = 2$

$300 = 100\left(1+\frac{0.08}{2}\right)^{2t}$

$3 = (1.04)^{2t}$

$\log_{10}(3) = \log_{10}\left[(1.04)^{2t}\right]$

$\log_{10}(3) = 2t\log_{10}(1.04)$

$\therefore t = \frac{\log_{10}(3)}{2\log_{10}(1.04)} \doteq \boxed{14 \text{ years}}$

[3]

Question 8: \$200 is deposited at the end of each month into a retirement fund earning 5% compounded monthly. How much is in the fund at the end of 45 years? (Round your final answer to the nearest dollar.)

200 200 200 ... 200 200

month: 0 1 2 3 ... $(45)(12) = 540$

$$A = P\left[\frac{(1+i)^m - 1}{i}\right]$$

where $P = 200$

$i = \frac{0.05}{12}$

$m = 540$

$$\therefore A = 200\left[\frac{\left(1+\frac{0.05}{12}\right)^{540} - 1}{\left(\frac{0.05}{12}\right)}\right]$$

$$\boxed{A \doteq \$405{,}287}$$

[5]

Question 9: An investor wishes to accumulate \$1,000,000 over 40 years by making annual payments into a fund earning 7% compounded annually. The payments for the first 20 years will be of size P while those of the second 20 years will be of size $2P$. Determine P. (Round your final answer to 2 decimal places.)

P P ... P 2P 2P ... 2P 2P

year: 0 1 2 ... 20 21 22 ... 39 40

After first 20 payments of size P:

$$A_1 = P\left[\frac{(1+0.07)^{20} - 1}{0.07}\right]$$

Future value of these first 20 payments at end of 40 years is

$$A_1(1+0.07)^{40-20} = P\left[\frac{(1.07)^{20} - 1}{(0.07)}\right](1.07)^{20}$$

Future value at time 40 years of the second 20 payments of size $2P$ is

$$A_2 = 2P\left[\frac{(1+0.07)^{20} - 1}{0.07}\right]$$

We require

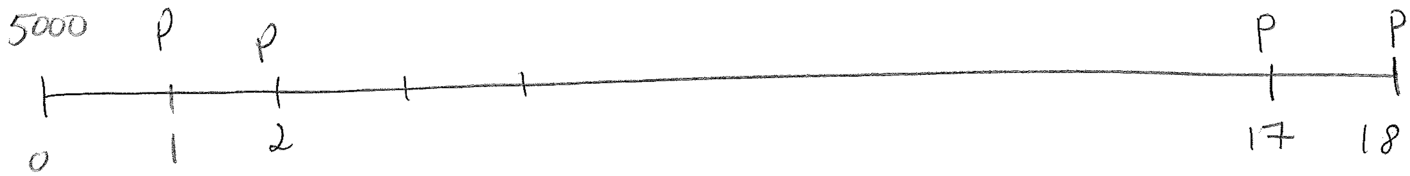
$$P\left[\frac{(1.07)^{20} - 1}{0.07}\right](1.07)^{20} + 2P\left[\frac{(1.07)^{20} - 1}{0.07}\right] = 1000\,000$$

$$\therefore P = \frac{1000\,000}{\left[\frac{(1.07)^{20} - 1}{0.07}\right]\left[(1.07)^{20} + 2\right]}$$

$$P \doteq \boxed{\$4155.75}$$

[5]

Question 10: Parents of a newborn wish to save \$30,000 by the child's eighteenth birthday to pay for college. All deposits will be made to a fund earning 8% compounded annually. If \$5000 is deposited when the child is born, what must be the deposits made at the end of each year to reach the saving goal? (Round your final answer to the nearest dollar.)



Future value of 5000 at end of 18 yrs:

$$A_1 = 5000\,(1+0.08)^{18} = 5000\,(1.08)^{18}.$$

Future value of payments of size P:

$$A_2 = P\left[\frac{(1+0.08)^{18}-1}{0.08}\right]$$

We require $A_1 + A_2 = 30000$

$$5000(1.08)^{18} + P\left[\frac{(1.08)^{18}-1}{0.08}\right] = 30000$$

So

$$P = \frac{30000 - 5000(1.08)^{18}}{\left[\frac{(1.08)^{18}-1}{0.08}\right]}$$

$$\boxed{P \doteq \$268.}$$

[5]

Question 11: A worker begins his new job on January 1 and makes \$3000 deposits to his pension fund every six months for his working lifetime. When he retires he has \$1,097,642 in his fund. If the fund earns 6% compounded semi-annually and the worker works a whole number of years, how many years did he work? (Round your answer to the nearest year.)

$$A = P\left[\frac{(1+i)^m - 1}{i}\right]$$

$A = 1{,}097{,}642$

$P = 3000$

$i = \frac{0.06}{2} = 0.03$

$$iA = P\left[(1+i)^m - 1\right]$$

$$\frac{iA}{P} + 1 = (1+i)^m$$

$$\log_{10}\left(\frac{iA}{P}+1\right) = \log_{10}(1+i)^m$$

$$\log_{10}\left(\frac{iA}{P}+1\right) = m\log_{10}(1+i)$$

$$m = \frac{\log_{10}\left(\frac{iA}{P}+1\right)}{\log_{10}(1+i)}$$

$$m = \frac{\log_{10}\left(\frac{(0.03)(1{,}097{,}642)}{3000}+1\right)}{\log_{10}(1.03)}$$

$m = 84$ payments, so he worked $\frac{84}{2} = \boxed{42 \text{ years}}$

[5]