**Question 1:** Use matrix reduction (Gaussian or Gauss-Jordan elimination) to solve the following system of equations:

$$6x + y = 8$$
$$x - 3y = -5$$
$$2x + y = \cancel{4}\,2$$

$$\left[\begin{array}{cc|c} 6 & 1 & 8 \\ 1 & -3 & -5 \\ 2 & 1 & 2 \end{array}\right]$$

$r_1 \leftrightarrow r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 6 & 1 & 8 \\ 2 & 1 & 2 \end{array}\right]$

$R_2 = (-6)r_1 + r_2$:
$R_3 = (-2)r_1 + r_3$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 19 & 38 \\ 0 & 7 & 12 \end{array}\right]$

$\to R_2 = \left(\frac{1}{19}\right)r_2$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 7 & 12 \end{array}\right]$

$R_3 = (-7)r_2 + r_3$: $\left[\begin{array}{cc|c} 1 & -3 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & -2 \end{array}\right]$

$\therefore$ System has no solutions

**[5]**

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**Question 2:** Use matrix reduction (Gaussian or Gauss-Jordan elimination) to solve the following system of equations:

$$x - 2y + 4z = -5$$
$$y - 2z = 1$$
$$-2y + 4z = -2$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 4 & -5 \\ 0 & 1 & -2 & 1 \\ 0 & -2 & 4 & -2 \end{array}\right]$$

$R_1 = 2r_2 + r_1$:
$R_3 = 2r_2 + r_3$: $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\to$ So $x = -3$

$y - 2z = 1 \Rightarrow y = 1 + 2z$

$\therefore$ solutions are $(-3, 1+2z, z)$ where $z$ is any real number

**[5]**

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**Question 3:** For this problem use the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 3 & -4 \\ 1 & 4 \\ 5 & -2 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

**(a)** Compute $(3\mathbf{A} - 4\mathbf{C})\mathbf{D}$, if possible. If the operation is not defined state "not defined".

$$(3A-4C)D = \left(3\begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 3 & -4 \\ 1 & 4 \\ 5 & -2 \end{bmatrix}\right)\begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 3 & 0 \\ 6 & 12 \\ -3 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -16 \\ 4 & 16 \\ 20 & -8 \end{bmatrix}\right)\begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 16 \\ 2 & -4 \\ -23 & 14 \end{bmatrix}\begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 70 \\ -16 \\ 152 \end{bmatrix}}$$

**[4]**

**(b)** Compute $\mathbf{AB} - 3\mathbf{I}_3$, if possible. If the operation is not defined state "not defined".

$$\begin{bmatrix} 1 & 0 \\ 2 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 4 & -3 & 0 \\ 1 & 1 & -2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -3 & 0 \\ 12 & -2 & -8 \\ -2 & 5 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & -3 & 0 \\ 12 & -5 & -8 \\ -2 & 5 & -7 \end{bmatrix}}$$

**[4]**

**(c)** Suppose there is some matrix **P** such that the product **APB** is defined. What must be the dimension of the matrix **P**?

$$\begin{matrix} A & P & B \\ 3\times 2 & 2\times 2 & 2\times 3 \end{matrix}$$

$\boxed{\therefore \text{ P must be } 2\times 2}$

**[2]**

**Question 4:**

**(a)** Determine $\mathbf{A}^{-1}$ where $\mathbf{A}$ is the matrix

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$R_2 = r_1 + r_2$ ;
$R_3 = (-1)r_1 + r_3$ ;

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 1 & 2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 = r_2 + r_1$ ;
$R_3 = (-1)r_2 + r_3$ ;

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & -1 & -2 & -1 & 1 \end{array}\right]$$

$R_3 = (-1)r_3$ ;

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 3 & 2 & 1 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$R_1 = (-3)r_3 + r_1$ ;
$R_2 = (-3)r_3 + r_2$ ;

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -2 & 3 \\ 0 & 1 & 0 & -5 & -2 & 3 \\ 0 & 0 & 1 & 2 & 1 & -1 \end{array}\right]$$

$$\boxed{\therefore A^{-1} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}}$$

**[7]**

**(b)** Use your result in part (a) to solve the following system of equations:

$$x - y = -1$$
$$-x + 2y + 3z = -3$$
$$x + 2z = 5$$

System is

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}}_{A \text{ above}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix}$$

So

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1} \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -3 \\ 5 \end{bmatrix} = \begin{bmatrix} 25 \\ 26 \\ -10 \end{bmatrix}$$

$$\boxed{\therefore x = 25, \ y = 26, \ z = -10.}$$

**[3]**

**Question 5:**

**(a)** Graph the following system of linear inequalities. Use the graph paper on the following page and keep your work tidy. Clearly label the various lines on your graph and make your graph large enough to show the important details.

$$y - x \leq 0$$
$$x + 2y \leq 10$$
$$5x + 4y \leq 40$$
$$x \geq 0$$
$$y \geq 0$$

| Inequality | Line | Test Pt | Test Result |
|---|---|---|---|
| $y-x \leq 0$ | $y-x=0$ | $(0,1)$ | $1-0 \overset{?}{\leq} 0$ : F |
| $x+2y \leq 10$ | $x+2y=10$ | $(0,0)$ | $0+2(0) \overset{?}{\leq} 10$ : T |
| $5x+4y \leq 40$ | $5x+4y=40$ | $(0,0)$ | $5(0)+4(0) \overset{?}{\leq} 40$ : T |

**[6]**

**(b)** Determine all corner points on your graph.

- By Inspection: $(0,0)$, $(8,0)$

- Solving:

① $x+2y=10$
② $5x+4y=40$

①: $x=10-2y$

②: $5(10-2y)+4y=40$
$50-10y+4y=40$
$-6y=-10$
$y=\frac{5}{3}$, $\therefore x=10-2\left(\frac{5}{3}\right)$
$= 20/3$

$\therefore \left(\frac{20}{3}, \frac{5}{3}\right)$

① $y-x=0$
② $x+2y=10$
①: $y=x$
②: $y+2y=10$
$3y=10$
$y=\frac{10}{3}$,
$\therefore x=\frac{10}{3}$

$\therefore \left(\frac{10}{3}, \frac{10}{3}\right)$

**[4]**

Graph for **Question 5:**