Question 1:

(a) Find the slope of the line through the points $(5, -4)$ and $(1, 3)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-4)}{1-5} = \boxed{\frac{-7}{4}}$$

[2]

(b) Determine the y-intercept of the line $3x + 2y = 13$.

$$3x + 2y = 13$$
$$2y = -3x + 13$$
$$y = -\frac{3}{2}x + \frac{13}{2}$$

so y-intercept is $\boxed{\frac{13}{2}}$

[2]

(c) Find an equation of the vertical line through the point $(-6, 5)$.

so $\boxed{x = -6}$

[2]

(d) Find k so that the line through $(4, -1)$ and $(k, 2)$ is parallel to $2x + 3y = 6$.

$$2x + 3y = 6$$
$$3y = -2x + 6$$
$$y = \frac{-2}{3}x + 2$$
$$\therefore \quad m = \frac{-2}{3}$$

so slope of line through $(4,-1)$ and $(k,2)$ must be $-\frac{2}{3}$:

$$\frac{2-(-1)}{k-4} = -\frac{2}{3}$$
$$3(3) = -2(k-4)$$
$$9 = -2k + 8$$
$$1 = -2k$$
$$\therefore \quad \boxed{k = -\frac{1}{2}}$$

[4]

Question 2: For each of the following pairs of lines, determine whether the lines are intersecting, parallel or coincident. If intersecting find the point of intersection. If parallel or coincident, give reasons for your answer:

(a)

$$\begin{aligned} L: &\quad 4x + 3y = 2 \\ M: &\quad 2x - \ y = 1 \end{aligned}$$

$M \Rightarrow y = 2x - 1$

$L \Rightarrow 4x + 3(2x-1) = 2$

$4x + 6x - 3 = 2$

$10x = 5$

$x = \frac{5}{10} = \frac{1}{2}$

So $y = 2x - 1$
$= 2(\frac{1}{2}) - 1$
$= 0$

$\therefore$ L & M intersect at $(x,y) = (\frac{1}{2}, 0)$

[4]

(b)

$$\begin{aligned} L: &\quad 4x - 2y = -7 \\ M: &\quad -2x + \ y = -2 \end{aligned}$$

$L: \quad y = 2x + \frac{7}{2}$

$M: \quad y = 2x - 2$

Equal slopes but different y-intercepts, so lines are parallel.

[3]

(c)

$$\begin{aligned} L: &\quad 2x + 3y = -7 \\ M: &\quad -4x - 6y = 14 \end{aligned}$$

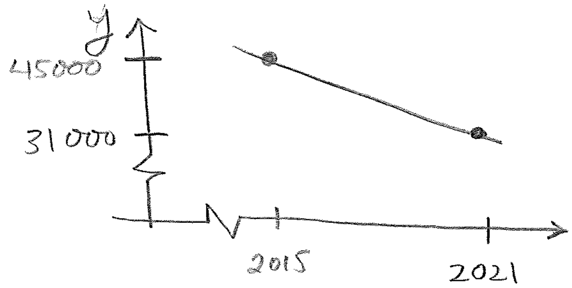
$L: \quad y = -\frac{2}{3}x - \frac{7}{3}$

$M: \quad y = -\frac{2}{3}x - \frac{7}{3}$

Same slopes and y-intercepts: lines are coincident.

[3]

Question 3: A person buys a new car on January 1, 2015 for \$45,000 and sells it exactly six years later for \$31,000. Assuming the value of the car decreases linearly (that is, following a straight line relationship), what was the value of the car four years after it was first purchased? (Round your final answer to the nearest dollar.)



$$m = \frac{y_2 - y_1}{t_2 - t_1} = \frac{45000 - 31000}{2015 - 2021} = -\frac{7000}{3}$$

Using $(t, y) = (2015, 45000)$ and $m = \frac{-7000}{3}$:

$$y - 45000 = -\frac{7000}{3}(t - 2015).$$

When $t = 2015 + 4 = 2019$,

$$y - 45000 = \frac{-7000}{3}(2019 - 2015)$$

So $y = 45000 - \frac{7000}{3}(4) \doteq \$35{,}667$

$\therefore$ Car is worth \$35,667 four years after purchase

[5]

Question 4: An investor has \$20,000 to invest and two investments are available. The first pays 4% per year while a second riskier investment pays 6% per year. The investor's goal is to earn and withdraw \$1075 from the investment fund each year. How much interest is earned from the 4% investment during the first year? Round your answer to 2 decimals.

Let x = amount invested at 4%
y = amount invested at 6%.

Then $x + y = 20000$ ①

$0.04x + 0.06y = 1075$ ②.

① $\Rightarrow y = 20000 - x$

② $\Rightarrow 0.04x + 0.06(20000 - x) = 1075$

$\therefore -0.02x + 1200 = 1075$

$-0.02x = 1075 - 1200$

$x = \frac{-125}{-0.02} = 6250$

$\therefore$ interest earned from 4% investment is $(0.04)(6250) = \$250$.

\$250 was earned from 4% investment

[5]

Question 5: The demand equation for a particular product is $D = 800 - 4p$ where p is price in dollars. At a price of \$100 the amount supplied is 1000 units. Determine the supply equation if the market price is \$75 .

At $p=100$, $S=1000$, so $(100, 1000)$ is a point on S-line.

At $p=75$, $D=800-4(75)=500$, so $(75, 500)$ is on D-line, but since $p=75$ is the market price, $(75, 500)$ is also on S-line.

Thus the supply equation is the line through $(100,1000)$ & $(75,500)$:

$$m = \frac{S_2 - S_1}{p_2 - p_1} = \frac{1000-500}{100-75} = 20.$$

$$S - S_1 = m(p - p_1)$$

$$\boxed{S - 1000 = 20(p-100)} \quad \underline{\text{or}} \quad \boxed{S = 20p - 1000}$$

[5]

Question 6: The library photocopier has monthly fixed costs of \$160 and it costs the library a further \$0.02 for each copy made. The library charges \$0.06 for each copy made. How many copies must be made each month in order for the library to break even on the photocopier costs?

Let x = number of copies made in a month.

$$C = 160 + 0.02x$$

$$R = 0.06x$$

$$C=R \implies 160 + 0.02x = 0.06x$$

$$\therefore \quad 0.04x = 160$$

$$x = \frac{160}{0.04} = 4000.$$

$\boxed{\therefore \text{ 4000 copies must be made each month}}$

[5]

Question 7: Solve the following system of equations **using Gaussian elimination** (no credit will be given for using any other method). Use proper notation to clearly state the row operations used at each step and clearly state the final solution.

$$\begin{aligned} 2x - 3y + 4z &= -15 \\ x - y + z &= -4 \\ 5x + y - 2z &= 12 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & -15 \\ 1 & -1 & 1 & -4 \\ 5 & 1 & -2 & 12 \end{array}\right]$$

$r_1 \leftrightarrow r_2$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{array}\right]$$

$R_2 = (-2)r_1 + r_2$:
$R_3 = (-5)r_1 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & 6 & -7 & 32 \end{array}\right]$$

$R_2 = (-1)r_2$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 6 & -7 & 32 \end{array}\right]$$

$R_3 = (-6)r_2 + r_3$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 5 & -10 \end{array}\right]$$

$R_3 = \left(\frac{1}{5}\right)r_3$:
$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$\therefore z = -2$

$y - 2z = 7$

$y - 2(-2) = 7$

$y = 3$

$x - y + z = -4$

$x - 3 - 2 = -4$

$x = 1$

$\boxed{\therefore (x, y, z) = (1, 3, -2)}$

[10]