Math 111 - Finite Math I

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Systems of Linear Equations

Systems of Linear Equations

A **system of linear equations** is a finite collection of linear equations:

Example: A system of 2 equations in 3 variables, or 3 unknowns:

$$\begin{aligned} 4x + 3y - 6z &= -2 \\ -x + y + z &= 2 \end{aligned}$$

A General System

A general system of m equations in n unknowns:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ \vdots \qquad\quad \vdots \qquad\qquad\qquad \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

A **solution** to this system is a set of numbers $x_1 = p_1, x_2 = p_2, \ldots, x_n = p_n$ which make all equations of the system true simultaneously.

Sometimes solutions are stated in **vector** form $(p_1, p_2, \ldots, p_n)$, also called an **ordered n-tuple**.

Consistent Systems of Equations

A system of linear equations is called **consistent** if it has **at least one solution**, otherwise it is called **inconsistent**.

Example: We saw that

$$\begin{aligned} 2x + 3y &= -4 \\ -3x + y &= -5 \end{aligned}$$

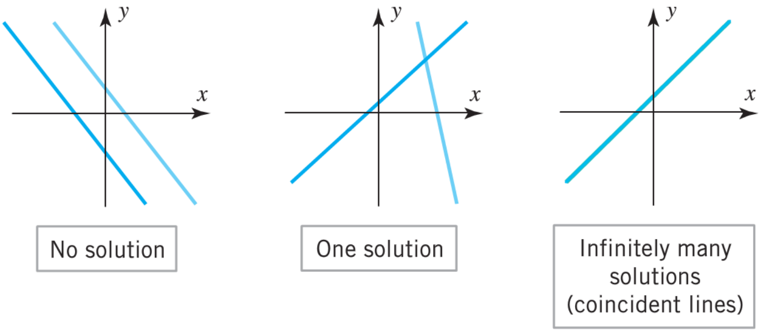
had solution $(1, -2)$, so the system is consistent.

Geometrically this says that the two lines intersect at the point $(1, -2)$.

Two Equations in Two Unknowns

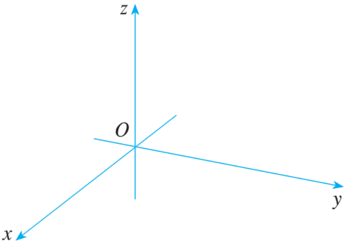
A linear equation in 2-variables represents a line in 2-dimensions.

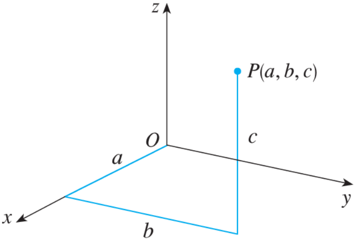
Two equations in two unknowns corresponds to lines that are either parallel, intersecting, or coincident:



Linear Equations in Three Variables

To graph an equation in 3-variables requires the three dimensional rectangular coordinate system:





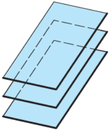
Planes in 3-Dimensions

The graph (set of all points satisfying the equation) of a linear equation in 3-variables is a **plane**, an infinite flat sheet.

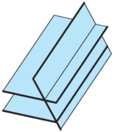
Example: Graph the plane $x + 2y + 3z = 6$

Three Equations in Three Unknowns

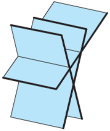
The solution of a system of three equations in three unknowns describes how the three planes intersect in three dimensions:



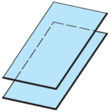
No solutions
(three parallel planes;
no common intersection)



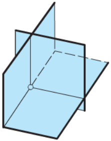
No solutions
(two parallel planes;
no common intersection)



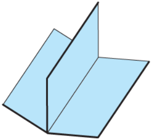
No solutions
(no common intersection)



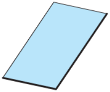
No solutions
(two coincident planes
parallel to the third;
no common intersection)



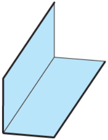
One solution
(intersection is a point)



Infinitely many solutions
(intersection is a line)



Infinitely many solutions
(planes are all coincident;
intersection is a plane)



Infinitely many solutions
(two coincident planes;
intersection is a line)

Important Observation

Notice that in the case of linear systems in two or three variables, **the systems have either zero, exactly one, or infinitely many solutions**. This turns out to be true for any system of linear equations.