Question 1: A flat circular plate occupies the region $x^2 + y^2 \leq 1$. The temperature at point (x, y) is given by $T(x, y) = x^2 + 2y^2 - x$. Determine the hottest and coldest temperatures on the plate.

[10]

Question 2: Find three real numbers whose sum is 9 and the sum of whose squares is as small as possible. Be sure to justify that the solution you find does indeed correspond to the desired minimum.
(You may know the answer to this immediately based on intuition or symmetry, but prove the result using calculus.)

[10]

Question 3: Find the volume of the solid that lies between the surface $z = y + \frac{x}{y^2}$ and the rectangle in the xy-plane $R = [0,2] \times [1,2]$.

[5]

Question 4: Evaluate the following double integral:

$$\int_0^1 \int_0^{y^2} y^3 e^{xy} \, dx \, dy$$

[5]

Question 5: Evaluate the following double integral:

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$$

(Hint: consider reversing the order of integration.)

[5]

Question 6: Find the volume of the region bounded between the paraboloid $z = x^2 + y^2$ and the triangle in the xy-plane enclosed by the lines $y = x$, $x = 0$ and $x + y = 2$.

[5]

Question 7: Compute

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$$

[5]

Question 8: Determine the volume in the first octant of the region lying above the cone $z=\sqrt{x^2+y^2}$ but below the top half of the sphere $x^2+y^2+z^2=32$.

[5]