Question 1:

(a) Determine the linear approximation $T_1(x)$ for $f(x) = e^{-x} + e^{-2x}$ at $a = 0$ and use it to approximate $f(0.2)$. Simplify your final answer.

[5]

(b) Give an error bound on your approximation in part (a). Again, simplify your final answer.

[5]

Question 2:

(a) Use a Taylor polynomial of degree 2 to approximate $1/1.1$. Simplify your final answer.

[5]

(b) Give an error bound on your approximation in part (a). Again, simplify your final answer.

[5]

Question 3:
Find the Taylor series about $a = -2$ for $f(x) = 1 + x + 2x^2 + 3x^3$. You should be able to write all terms of the series.

[5]

Question 4: Find the first four nonzero terms of the Taylor series about $a = 1$ for $g(x) = \dfrac{3}{2 + 4x}$ and state the open interval of convergence.

[5]

Question 5: Find the Maclaurin polynomial of degree 7 for $f(x) = \dfrac{\ln(1+x^2)}{x}$.

[3]

Question 6: Find the sum of the infinite series $1 - \dfrac{\pi^2}{2!} + \dfrac{\pi^4}{4!} - \cdots$.

[3]

Question 7: Find the first three non-zero terms of the Maclaurin series for $f(x) = e^{-x}\ln(1+x)$.

[4]

Question 8: Use series to find the limit: $\displaystyle\lim_{x\to 0}\frac{\sin x - x + \dfrac{x^3}{6}}{x^5}$

[5]

Question 9: It can be shown that

$$\cos\left[\sin\left(x\right)\right] = 1 - \frac{x^2}{2} + \frac{5x^4}{24} - \frac{37x^6}{720} + \cdots$$

for all real numbers x. Use this fact to find the first 3 non-zero terms of the Maclaurin series for $g(x) = \sin\left[\sin\left(x^2\right)\right]\,\cos\left(x^2\right)$.

[5]