Question 1: Evaluate the following limits if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate limits.)

(a) $\lim\limits_{x \to -2} \dfrac{2 - |x|}{2 + x}$

[3]

(b) $\lim\limits_{x \to 0} \dfrac{\sin(3x)\sin(5x)}{x^2 \cos(x)}$

[3]

(c) $\lim\limits_{x \to 2\pi^-} x \csc(x)$

[4]

Question 2: Find $\lim_{x \to 0} x^2 \cos(\pi / x^5)$.
(Hint: Squeeze Theorem. Be sure to demonstrate that all conditions of the theorem are satisfied, and state a clear conclusion which references use of the theorem.)

[5]

Question 3: Let

$$f(x) = \begin{cases} cx^2 + 2x & \text{if } x < 2 \\ x^3 - cx & \text{if } x \geq 2 \end{cases}$$

Find the constant c that makes f continuous at all real numbers.

[5]

Question 4: Let $f(x) = \dfrac{1}{x + \sqrt{x}}$. Use the Intermediate Value Theorem to show that $f(x) = 1/3$ for some x in the interval $[1, 4]$.

[5]

Question 5: Determine $\lim\limits_{t \to \infty} \dfrac{\sqrt{t} + t^2}{2t - t^2}$.

If the limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate the limit.)

[5]

Question 6:

(a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{2-x}$. Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

[7]

(b) Use your result from part (a) to find the equation of the tangent line to the curve $y = x/(2-x)$ at the point where $x = -1$.

[3]

Question 7: Differentiate (that is, find the derivative of) the following functions.

(a) $f(x) = 2x^3 - 3x^2 + 2x + 4$

[2]

(b) $y = 6\sqrt{x} + \dfrac{1}{\sqrt{x}}$

[2]

(c) $g(t) = (3t+1)^2 - \dfrac{\cos(t)}{\pi}$

[2]

Question 8: Suppose $c \neq 0$ and that the tangent line to $y = x^2$ at $x = c$ has x-intercept 2. Determine c.

[4]