Question 1: Simplify: $\frac{2}{a^2} - \frac{3}{ab} + \frac{4}{b^2}$ $= \boxed{\frac{2b^2 - 3ab + 4a^2}{a^2 b^2}}$

[3]

Question 2: Solve for x: $2x^2 + 7x = 4$

$2x^2 + 7x - 4 = 0$

$2x^2 + 8x - x - 4 = 0$

$2x(x+4) - (x+4) = 0$

$(2x-1)(x+4) = 0$

$2x - 1 = 0$

$\boxed{x = \frac{1}{2}}$ | $\boxed{x = -4}$

[3]

Question 3: Simplify: $\frac{x}{x^2 + x - 2} - \frac{2}{x^2 - 5x + 4}$

$= \frac{x}{(x-1)(x+2)} - \frac{2}{(x-1)(x-4)}$

$= \frac{x(x-4) - 2(x+2)}{(x-1)(x+2)(x-4)}$

$= \boxed{\frac{x^2 - 6x - 4}{(x-1)(x+2)(x-4)}}$

[4]

Question 4: Simplify: $\dfrac{\sqrt[5]{96a^6}}{\sqrt[5]{3a}}$ $= \sqrt[5]{\dfrac{96a^6}{3a}}$

$= \sqrt[5]{32a^5}$

$= \boxed{2a}$

[3]

Question 5: Rationalize: $\dfrac{\sqrt{x^2+3x+4}-x}{1} \cdot \dfrac{\sqrt{x^2+3x+4}+x}{\sqrt{x^2+3x+4}+x}$

$= \dfrac{x^2+3x+4-x^2}{\sqrt{x^2+3x+4}+x}$

$= \boxed{\dfrac{3x+4}{\sqrt{x^2+3x+4}+x}}$

[4]

Question 6: Find an equation of the line passing through the point $(2, -7)$ which is perpendicular to the line $2x + 5y - 8 = 0$.

$2x+5y-8=0$

$y = -\frac{2}{5}x + \frac{8}{5}$

$\therefore$ $m = \frac{-2}{5}$

$\therefore$ slope of perpendicular line is $m_\perp = \frac{5}{2}$

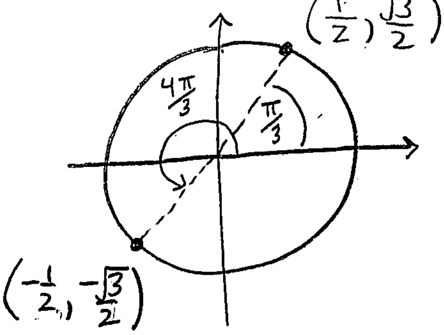
$\therefore$ Equation is

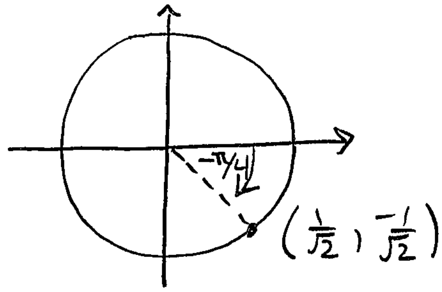
$y - y_0 = m_\perp (x - x_0)$

$\boxed{y+7 = \frac{5}{2}(x-2)}$

[3]

Question 7: Determine $\tan(4\pi/3) - \cos(-\pi/4)$





$\therefore \tan\left(\frac{4\pi}{3}\right) - \cos\left(-\frac{\pi}{4}\right)$

$= \sqrt{3} - \frac{1}{\sqrt{2}}$

$= \boxed{\frac{\sqrt{6}-1}{\sqrt{2}}}$

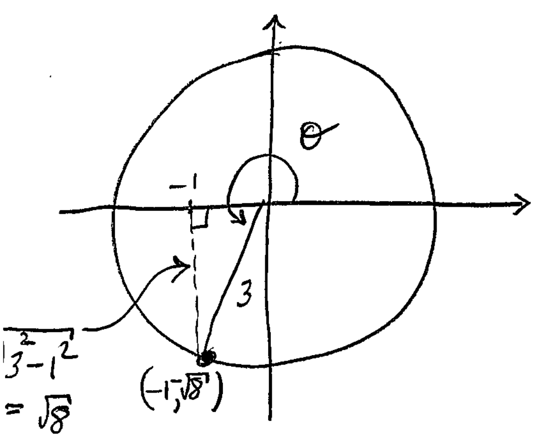
$\therefore \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\therefore \tan\left(\frac{4\pi}{3}\right) = \frac{\left(\frac{-\sqrt{3}}{2}\right)}{\left(\frac{-1}{2}\right)} = \sqrt{3}$

[3]

Question 8: If $\cos(\theta) = -1/3$ where $\pi < \theta < 3\pi/2$ then determine $\csc(\theta)$

$\cos(\theta) = \frac{x}{r} = \frac{-1}{3}$



$\therefore \csc(\theta) = \frac{r}{y} = \boxed{\frac{-3}{\sqrt{8}}}$

[3]

Question 9: Find all values of x in the interval $[0, 2\pi]$ for which $2\sin^2(x) + \sin(x) = 1$.

$2\sin^2(x) + \sin(x) - 1 = 0$

$\sin(x) = \frac{-1 \pm \sqrt{1^2 - 4(2)(-1)}}{2(2)}$

$\sin(x) = \frac{-1 \pm 3}{4}$

$\sin(x) = -1; \quad \sin(x) = \frac{1}{2}$

$\therefore x = \frac{3\pi}{2} \quad ; \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\boxed{\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}}$

[4]

Question 10: Determine the domain of $f(x) = \sqrt{3-x}\,\sin\left(\frac{1}{\sqrt{x-1}}\right)$.

We require $3-x \geq 0$ and $x-1 > 0$

$\Rightarrow$ $x \leq 3$ and $x > 1$

$\therefore$ $1 < x \leq 3$

so domain is $(1, 3]$.

[3]

Question 11: Find functions f, g and h so that $f(g(h(x))) = \frac{4}{1+\sqrt{x-1}}$.
(There are several possible correct answers. Do not let $h(x) = x$.)

Let $h(x) = x-1$ or $h(x) = \sqrt{x-1}$

$g(x) = \sqrt{x}$ $g(x) = 1+x$

$f(x) = \frac{4}{1+x}$ $f(x) = \frac{4}{x}$

[3]

Question 12: Let $f(x) = x+4$ and $h(x) = 4x-1$. Find a function g so that $g \circ f = h$.

$\boxed{g(x) = 4x - 17}$

[4]

Question 13: Evaluate the following limit, if it exists: $\lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \begin{matrix} \to \text{"}0\text{"} \\ \to 0 \end{matrix}$

$$= \lim_{x \to 1} \frac{x-1}{\sqrt{x+3}-2} \cdot \frac{\sqrt{x+3}+2}{\sqrt{x+3}+2}$$

$$= \lim_{x \to 1} \frac{(x-1)\sqrt{x+3}+2}{x+3-4}$$

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(\sqrt{x+3}+2)}{\cancel{(x-1)}}$$

$$= \boxed{4}$$

[3]

Question 14: Evaluate the following limit, if it exists: $\lim_{x \to -3} \frac{x+3}{x^2+4x+3} \begin{matrix} \to \text{"}0\text{"} \\ \to 0 \end{matrix}$

$$= \lim_{x \to -3} \frac{\cancel{(x+3)}}{\cancel{(x+3)}(x+1)}$$

$$= \boxed{\frac{-1}{2}}$$

[3]

Question 15: Evaluate the following limit, if it exists: $\lim_{x \to 0} \frac{\frac{1}{x-1} + \frac{1}{x+1}}{x} \begin{matrix} \to \text{"}0\text{"} \\ \to 0 \end{matrix}$

$$= \lim_{x \to 0} \frac{1}{x}\left(\frac{1}{x-1} + \frac{1}{x+1}\right)$$

$$= \lim_{x \to 0} \frac{1}{x}\left(\frac{x+\cancel{1}+x-\cancel{1}}{(x-1)(x+1)}\right)$$

$$= \lim_{x \to 0} \frac{1}{\cancel{x}} \frac{2\cancel{x}}{(x-1)(x+1)}$$

$$= \boxed{-2}$$

[4]