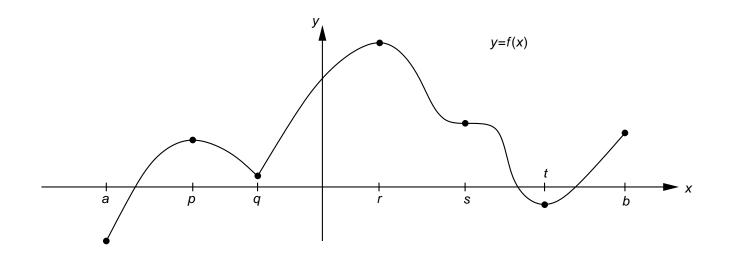
We continue our work on using derivatives to study graphs of functions. So far we have seen how derivatives can be used to identify the absolute extrema of a continuous function on a closed interval. In this lesson we ask: What information does f'(x) give us about the shape of the graph of y = f(x)? Begin with some terminology. Consider the graph of a general function f with domain D = [a, b]:



Recall:

- A function f is said to be increasing on an interval if for any numbers x₁ < x₂ from the interval, f(x₁) < f(x₂). If a function is increasing on an interval then its graph rises as x increases.
 In the graph above f is increasing on [a, p], [q, r] and [t, b].
- ► A function f is said to be **decreasing** on an interval if for any numbers x₁ < x₂ from the interval, f(x₁) > f(x₂). If a function is decreasing on an interval then its graph falls as x increases. In the graph above f is decreasing on [p, q] and [r, t].

Using derivatives we can easily determine the intervals of increase and decrease of a function.

Increasing/Decreasing Test

Recall

$$f^{\prime}(c)=$$
 slope of the tangent line to graph of $y=f(x)$ at $x=c$,

so

 $f'(c) > 0 \Rightarrow$ outputs of f are increasing as x passes through c \Rightarrow graph of f is rising as x passes through c

$$f'(c) < 0 \Rightarrow$$
 outputs of f are decreasing as x passes through c

 \Rightarrow graph of f is falling as x passes through c

- (i) If f'(x) > 0 on an interval, then f is increasing on that interval.
- (ii) If f'(x) < 0 on an interval, then f is decreasing on that interval.

Observe on our graph: whenever f changes from increasing to decreasing, or vice versa, it does so at a critical number (x = p, q, r and t). However, not every critical number corresponds to such a change: f is decreasing on both sides of x = s. Putting all of this together:

To determine the intervals of increase and decrease of a function f:

- (i) Find points at which f' changes sign (from positive to negative or vice versa). f' can change sign at
 - critical numbers: x-values at which f'(x) = 0 or f'(x) does not exist
 - \circ values of x at which f itself is not defined
- (ii) Test f'(x) on the subintervals defined by the points from (i).

Once you have determined the intervals of increase/decrease of f, it is easy to read off the relative extrema (that is, the relative maxima and minima) using

The First Derivative Test: Suppose x = c is a critical number of a continuous function f.

- (i) If f' changes from positive to negative at x = c, then f has a relative maximum of f(c) at x = c.
- (ii) If f' changes from negative to positive at x = c, then f has a relative minimum of f(c) at x = c.
- (iii) If f' does not change sign at x = c, then f has a neither a relative maximum nor relative minimum at x = c.