Question 1: Find the following derivatives (Take care to use proper notation; it is not necessary to simplify your answers):

(a) $f(x) = 4x^3 \cos(x)$

$$\boxed{f'(x) = 12x^2 \cos(x) - 4x^3 \sin(x)}$$

[2]

(b) $y = \dfrac{\sqrt{x}}{1 + \tan(x)}$ $= \dfrac{x^{1/2}}{1+\tan(x)}$

$$\boxed{y' = \frac{[1+\tan(x)]\,\frac{1}{2}x^{-1/2} - x^{1/2}\sec^2(x)}{[1+\tan(x)]^2}}$$

[2]

(c) $g(t) = t^2 \sec(1/t)$

$$g'(t) = 2t\sec\left(\tfrac{1}{t}\right) + \cancel{t^2}\sec\left(\tfrac{1}{t}\right)\tan\left(\tfrac{1}{t}\right)\left(\frac{-1}{\cancel{t^2}}\right)$$

$$= \boxed{2t\sec\left(\tfrac{1}{t}\right) - \sec\left(\tfrac{1}{t}\right)\tan\left(\tfrac{1}{t}\right)}$$

[3]

(d) $y = \dfrac{x^2}{\left(1 + \frac{1}{x^2}\right)}$ $\cdot \dfrac{x^2}{x^2} = \dfrac{x^4}{1+x^2}$

$$y' = \frac{(1+x^2)\,4x^3 - x^4(2x)}{(1+x^2)^2}$$

$$= \boxed{\frac{4x^3 + 2x^5}{(1+x^2)^2}}$$

[3]

Question 2: Find the following derivatives (Take care to use proper notation; it is not necessary to simplify your answers):

(a) $f(x) = \cos^4(1-2x)$

$f'(x) = 4\cos^3(1-2x) \cdot [-\sin(1-2x)] \cdot (-2)$

$= \boxed{8\cos^3(1-2x)\sin(1-2x)}$

[2]

(b) $y = \frac{1}{2}\cos(2x)\sqrt{1+\sin(2x)}$ $= \frac{1}{2}\cos(2x)\,[1+\sin(2x)]^{1/2}$

$y' = -\frac{1}{\cancel{2}}\sin(2x)\cdot\cancel{2}\cdot[1+\sin(2x)]^{1/2} + \frac{1}{\cancel{2}}\cos(2x)\,\frac{1}{2}[1+\sin(2x)]^{-\frac{1}{2}}\cdot\cos(2x)\cdot\cancel{2}$

$= \boxed{-\sin(2x)[1+\sin(2x)]^{1/2} + \frac{1}{2}\cos^2(2x)\,[1+\sin(2x)]^{-\frac{1}{2}}}$

[2]

(c) $g(t) = t^{-2}(4+t^3)^7$

$g'(t) = -2t^{-3}(4+t^3)^7 + t^{-2}\cdot 7(4+t^3)^6(3t^2)$

$= \boxed{-2t^{-3}(4+t^3)^7 + 21(4+t^3)^6}$

[3]

(d) $y = \sqrt{\tan\left(\frac{x}{1+x}\right)}$ $= \left[\tan\left(\frac{x}{1+x}\right)\right]^{\frac{1}{2}}$

$y' = \frac{1}{2}\left[\tan\left(\frac{x}{1+x}\right)\right]^{-\frac{1}{2}}\cdot\sec^2\left(\frac{x}{1+x}\right)\cdot\frac{(1+\cancel{x})\cdot 1 - \cancel{x}\cdot 1}{(1+x)^2}$

$= \boxed{\dfrac{\sec^2\left(\frac{x}{1+x}\right)}{2\sqrt{\tan\left(\frac{x}{x+1}\right)}\,(1+x)^2}}$

[3]

Question 3: Find $\frac{dy}{dx}$ by implicit differentiation:

$$x^4 + \sin(y) = x^3y^2$$

$$\frac{d}{dx}\left[x^4 + \sin(y)\right] = \frac{d}{dx}\left[x^3y^2\right]$$

$$4x^3 + \cos(y)\,y' = 3x^2y^2 + x^3 \cdot 2yy'$$

$$y'\left[\cos(y) - 2x^3y\right] = 3x^2y^2 - 4x^3$$

$$\boxed{y' = \frac{3x^2y^2 - 4x^3}{\cos(y) - 2x^3y}}$$

[5]

Question 4: Find an equation of the tangent line to

$$y^4 - 4y^2 = x^4 - 9x^2$$

at the point $(-3, 2)$.

$$\frac{d}{dx}\left[y^4 - 4y^2\right] = \frac{d}{dx}\left[x^4 - 9x^2\right]$$

$$4y^3y' - 8yy' = 4x^3 - 18x$$

at $(-3,2)$:

$$4(2)^3y' - 8(2)y' = 4(-3)^3 - 18(-3)$$

$$16y' = -54$$

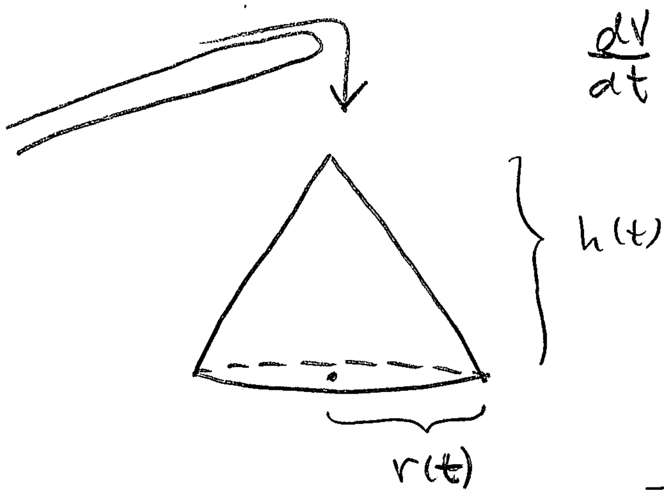
$$y' = \frac{-54}{16} = \frac{-27}{8}$$

$\therefore$ Equation of tangent line is

$$\boxed{y - 2 = \frac{-27}{8}(x+3)}$$

[5]

Question 5: Sand falls from a conveyor belt onto a cone shaped pile at the rate of 10 m^3/min. The pile of sand grows in such a way that the height of the pile is always 3/8 of the base diameter. How fast is the height of the pile growing when the pile is 4 m tall? (Recall, the volume of a cone of height h and base radius r is $V = \pi r^2 h/3$.)



$$\frac{dV}{dt} = +10 \frac{m^3}{min}$$

$$h(t) = \frac{3}{8}\underbrace{\left[2r(t)\right]}_{\text{diameter}}$$

$$\therefore\ h(t) = \frac{3}{4} r(t) \Rightarrow r(t) = \frac{4}{3} h(t)$$

Find $\frac{dh}{dt}$ when $h = 4$ m.

$$V = \frac{\pi}{3} r^2 h$$

$$= \frac{\pi}{3}\left[\frac{4}{3}h\right]^2 h$$

$$= \frac{16\pi}{27} h^3$$

$$\therefore\ \frac{dV}{dt} = \frac{16\pi}{\cancel{27}\,9} \cancel{3}h^2 \frac{dh}{dt}$$

when $h = 4$:

$$10 = \frac{16\pi}{9} \cdot 4^2 \cdot \frac{dh}{dt}$$

$$\therefore\ \frac{dh}{dt} = \frac{90}{4^4 \pi}$$

$$= \frac{45}{128\pi} \frac{m}{min.}$$

$\boxed{\therefore \text{ Height of pile is increasing at } \frac{45}{128\pi}\frac{m}{min}.}$

[10]

Question 6: For this question consider the function $f(x) = (1+x)^k$ where k is a constant.

(a) Find $L(x)$, the linearization of f at $a = 0$. (Your answer should have k in it.)

$f(x) = (1+x)^k$; $f(a) = (1+0)^k = 1$

$f'(x) = k(1+x)^{k-1}$; $f'(a) = k(1+0)^{k-1} = k$

$\therefore$ $L(x) = f(a) + f'(a)(x-a)$

$\boxed{L(x) = 1 + kx}$

[4]

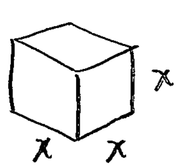
(b) Use part (a) to approximate $(1.1)^\pi$.

Here $(1+x)^k \approx (1.1)^\pi$, so $x = \frac{1}{10}$ and $k = \pi$.

$\therefore$ $(1.1)^\pi \approx 1 + \pi \cdot \left(\frac{1}{10}\right) = \boxed{1 + \frac{\pi}{10}}$

[1]

Question 7: The side length of a cube was measured to be 10 cm with a maximum measurement error of 1/100. Estimate the maximum <u>relative</u> error in the calculated surface area of the cube.



$x = 10$

$dx = \frac{1}{100}$

$S = 6x^2$.

Approximate relative error is

$$\frac{dS}{S} = \frac{\frac{d}{dx}[6x^2]\,dx}{6x^2} = \frac{12x\,dx}{6x^2} = \frac{2\,dx}{x} = \frac{2\left(\frac{1}{100}\right)}{10} = \frac{2}{1000} = \boxed{\frac{1}{500}}$$

[5]