Question 1: Let

$$W = \frac{v}{2u+v}$$

where

$$u = pq\sqrt{r} \quad \text{and} \quad v = p\sqrt{q}r$$

Determine $\dfrac{\partial W}{\partial q}$ when $p = 2$, $q = 1$ and $r = 4$.

[5]

Question 2: Let $T(x, y)$ be the temperature in degrees Celcius at point (x, y), and suppose a bug crawls so that its position after t seconds is given by $x = \sqrt{1+t}$ and $y = 2 + \frac{1}{3}t$ where x and y are measured in centimetres. If $T_x(2, 3) = 4$ and $T_y(2, 3) = 3$, how fast is the temperature rising along the bug's path after 3 seconds? State units with your answer.

[5]

Question 3: Find the directional derivative of $f(x, y) = x^2 e^{-y}$ at the point $(-2, 0)$ in the direction toward the point $(2, -3))$.

[5]

Question 4: Find the maximum rate of change of $f(x, y) = x^2 y + \sqrt{y}$ at the point $(2, 1)$ and the direction in which it occurs.

[5]

Question 5: Find the equation of the tangent plane to the surface

$$x^4 + y^4 + z^4 = 3x^2y^2z^2$$

at the point $(1, 1, 1)$.

[5]

Question 6: At what point(s) does the normal line through $(1, 2, 1)$ on the ellipsoid $4x^2 + y^2 + 4z^2 = 12$ intersect the sphere $x^2 + y^2 + z^2 = 102$?

[5]

Question 7: Find all critical points of $f(x, y) = (x^2 + y^2)e^{-x}$ and classify each as either a local maximum, local minimum or a saddle point. Carefully calculate all required derivatives and keep your work organized.

[10]

Question 8: Find the dimensions of a rectangular box (that is, a conventional box with six rectangular faces) having maximum volume if the sum of the lengths of the 12 edges is a constant c. (The edges of the box are the llines where the rectangular faces meet). Explain why your solution does indeed correspond to the absolute maximum volume.
Note: The solution to this problem can be deduced using a symmetry argument, but for this question you are required to prove the result using calculus.

[10]