

Question 1: A particle starting at initial position $s(0) = 0$ m and initial velocity of k m/s has acceleration at time t second of $a(t) = k\sqrt{t}$ m/s². Here k is a constant. Find the particle's position after 1 second.

[5]

Question 2: Use an area interpretation to find $\int_0^2 |3 - 2x| dx$

[5]

Question 3: Use the definition of the definite integral in the form

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

to evaluate

$$\int_{-2}^1 (1 - 2x^2) dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

Question 4: Find the average value of $f(x) = \pi(\sin(x) + \sec^2(x))$ over the interval $[-\pi/4, \pi/4]$. (State your answer as a single fraction.)

[5]

Question 5: A certain population is currently of size 20 individuals and is increasing at a rate of $r(t) = (1+t^2)/3$ individuals per week. How large will the population be in three weeks time?

[5]

Question 6:

(a) Evaluate $\int_4^9 \sqrt{x} - \frac{1}{\sqrt{x}} dx$.

[3]

(b) Evaluate $\int_0^1 x^{10} + 10^x dx$.

[3]

Question 7: Suppose f is a continuous function with the property that

$$\int_0^x f(t) dt = \sin(2x) - \int_0^x \cos(2t)f(t) dt .$$

Find a formula for $f(x)$.

[4]

Question 8: (Substitution Method)

(a) Determine $\int \frac{x}{(4x^2 + 1)^5} dx$.

[3]

(b) Evaluate $\int_1^{\sqrt{e}} \frac{\sin(\pi \ln x)}{x} dx$.

[3]

(c) Determine $\int \frac{e^{\sec(\sqrt{x})} \sec(\sqrt{x}) \tan(\sqrt{x})}{\sqrt{x}} dx$.

[4]