Question 1:

(a) Find the linear approximation $T_1(x)$ about $a = 2$ for $f(x) = \dfrac{1}{x+2}$.

[4]

(b) Use your result in part (a) to approximate $f(3/2)$. (Express your answer as a single simplified fraction.)

[2]

(c) Give an error bound for your approximation in part (b). (Again, express your answer as a single simplified fraction.)

[4]

Question 2:

(a) Use a Taylor (or Maclaurin) polynomial of degree 3 to approximate $e^{-0.1}$. (Express your answer as a single fraction; there is no need to reduce it to lowest terms.)

[5]

(b) Give an error bound on your approximation in part (a). (Express your answer as a single simplified fraction.)

[5]

Question 3:

(a) Find the first four nonzero terms of the Maclaurin series for $f(x) = \dfrac{\sin(x)}{1+x}$.

[4]

(b) Determine $f^{(4)}(0)$, the fourth derivative of f at 0, where $f(x)$ is as in part (a). Simplify your final answer.

[2]

Question 4: It can be shown that the Maclaurin series for $\sin^4(x)$ is

$$\sin^4(x) = x^4 - \frac{2x^6}{3} + \frac{x^8}{5} - \cdots$$

Use this to find the first three nonzero terms of the Maclaurin series for $g(x) = 4\sin^3(x)\cos(x)$.

[4]

Question 5: Evaluate the following limit:

$$\lim_{x\to 0} \frac{\ln\left(1+x^2\right) - e^{(x^2)} + 1}{2x^4}$$

[5]

Question 6: Find the the first four nonzero terms of the Taylor series for $h(x) = \dfrac{1}{2-3x}$ about $a = 1$ and state the open interval of convergence.

[5]

Question 7: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=1}^{\infty} \frac{4^k (x-3)^{2k}}{k^2}$$

[5]

Question 8: Find the radius of convergence R and open interval of convergence $\mathcal{I}$ for the power series

$$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k (x+1)^k}{2^k k!}$$

[5]