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Sum Law

- $\blacktriangleright \lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
- ▶ In words: The limit of a sum is the sum of the limits
- Example: $\lim_{x \to \pi} \left[\sqrt{x} + \sin x \right] = \left(\lim_{x \to \pi} \sqrt{x} \right) + \left(\lim_{x \to \pi} \sin x \right)$

Difference Law

- $\blacktriangleright \lim_{x \to a} [f(x) g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$
- In words: The limit of a difference is the difference of the limits
- Example: $\lim_{x \to -3} \left[\frac{1}{x} x^3 \right] = \left(\lim_{x \to -3} \frac{1}{x} \right) \left(\lim_{x \to -3} x^3 \right)$

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Constant Multiplier Law

- $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$
- In words: The limit of a constant times a function is the constant times the limit of the function.

• Example:
$$\lim_{x \to \sqrt{2}} \left[\frac{3}{7\sqrt{x}} \right] = \frac{3}{7} \left(\lim_{x \to \sqrt{2}} \frac{1}{\sqrt{x}} \right)$$

Product Law

$$\blacktriangleright \lim_{x \to a} [f(x)g(x)] = \left(\lim_{x \to a} f(x)\right) \left(\lim_{x \to a} g(x)\right)$$

► In words: The limit of a product is the product of the limits

• Example:

$$\lim_{x \to 0} \left[(x^2 + 2)(1 + \cos x) \right] = \left(\lim_{x \to 0} (x^2 + 2) \right) \left(\lim_{x \to 0} (1 + \cos x) \right)$$

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Quotient Law

- $\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided $\lim_{x \to a} g(x) \neq 0$.
- ► In words: The limit of a quotient is the quotient of the limits

• Example:
$$\lim_{x \to 0} \left[\frac{x^2 + 2}{1 + \cos x} \right] = \frac{\lim_{x \to 0} (x^2 + 2)}{\lim_{x \to 0} (1 + \cos x)}$$

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Power Law

- $\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$ where *n* is a positive integer.
- ► In words: The limit of a power is the power of the limit
- Example: $\lim_{x \to \pi} [x + \tan x]^{1000} = \left[\lim_{x \to \pi} (x + \tan x)\right]^{1000}$

Root Law

- $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$ where *n* is a positive integer, and where $\lim_{x \to a} f(x) > 0$ if *n* is even.
- ► In words: The limit of a root is the root of the limit
- Example: $\lim_{x \to 1} \sqrt{x^2 + 5x^3} = \sqrt{\lim_{x \to 1} (x^2 + 5x^3)}$

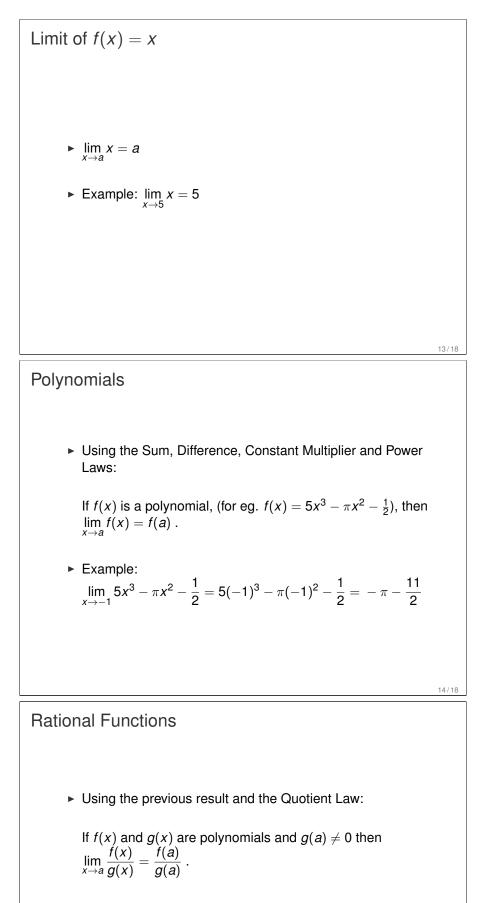
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Particular Limit Results

Constants

- $\blacktriangleright \lim_{x \to a} c = c$
- Example: $\lim_{x\to 3} \sqrt{2\pi} = \sqrt{2\pi}$

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► Example:
$$\lim_{x \to 2} \frac{2x^3 - x}{3x + 1} = \frac{2(2)^3 - (2)}{3(2) + 1} = \frac{14}{7} = 2$$

