Question 1: Simplify: $\frac{1}{x+5} - \frac{1}{x^2-25}$ $= \frac{x-5-1}{(x+5)(x-5)}$

$$= \boxed{\frac{x-6}{(x+5)(x-5)}}$$

[3]

Question 2: Rationalize the numerator and simplify: $\frac{\sqrt{x}-\sqrt{y}}{y-x}$ $\cdot \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

$$= \frac{x-y}{(y-x)(\sqrt{x}+\sqrt{y})}$$

$$= \frac{\cancel{(x-y)}\ 1}{-\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}$$

$$= \boxed{\frac{-1}{\sqrt{x}+\sqrt{y}}}$$

[4]

Question 3: Simplify and state your answer using only positive exponents: $\frac{x^{-1}+y^{-1}}{(x-y)^{-1}}$

$$\frac{x^{-1}+y^{-1}}{(x-y)^{-1}} \cdot \frac{(x-y)}{(x-y)} = \frac{\cancel{1} + xy^{-1} - x^{-1}y \cancel{-1}}{1}$$

$$= \frac{x}{y} - \frac{y}{x}$$

$$= \boxed{\frac{x^2-y^2}{xy}}$$

[3]

Question 4: The equation of the line through the points $(a, -3)$ and $(-1, -10)$ is $y = 7x - 3$. Determine the value of a.

Method 1:

$(a, -3)$ is on $y = 7x - 3$,

so $-3 = 7a - 3$

$\Rightarrow 0 = 7a$

$\therefore \boxed{a = 0}$

Method 2:

slope of $y = 7x - 3$ is 7, so

$7 = \frac{-10-(-3)}{-1-a}$

$7 = \frac{-7}{-1-a}$

$-1-a = \frac{-7}{7}$

$\therefore -1-a = -1$

$-a = 0$

$\boxed{a = 0}$

[3]

Question 5: Solve: $2x^2 = 7x - 2$

$2x^2 - 7x + 2 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(2)}}{2(2)}$

$= \frac{7 \pm \sqrt{33}}{4}$

$\therefore x = \frac{7 + \sqrt{33}}{4}$, $x = \frac{7 - \sqrt{33}}{4}$

[4]

Question 6: Find an equation of the line passing through the point $(1, 1)$ which is parallel to the line $3x + 2y - 7 = 0$.

$3x + 2y - 7 = 0$

$2y = -3x + 7$

$y = \left(-\frac{3}{2}\right)x + \frac{7}{2}$

$\therefore m = -\frac{3}{2}$

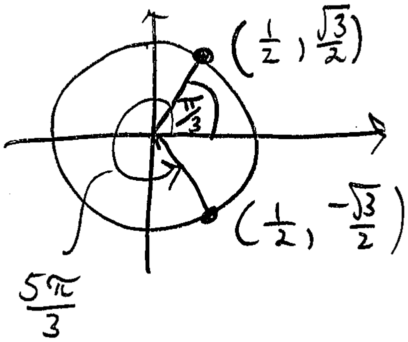
$y - y_0 = m(x - x_0)$

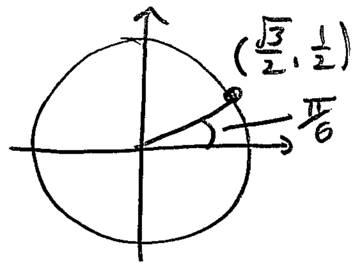
$\boxed{y - 1 = -\frac{3}{2}(x - 1)}$

or $\boxed{y = \frac{-3}{2}x + \frac{5}{2}}$

[3]

Question 7: Determine $\cos(5\pi/3) - \sin(\pi/6)$





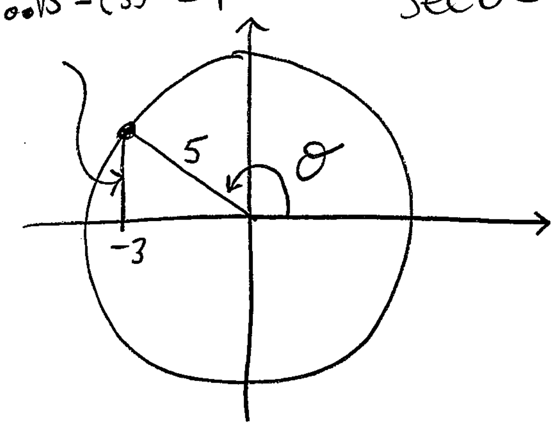
$\therefore \cos(5\pi/3) - \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} - \frac{1}{2} = \boxed{0}$

[3]

Question 8: If $\sec(\theta) = -5/3$ where $\pi/2 < \theta < \pi$ then determine $\sin(\theta)$

$\therefore \sqrt{5^2 - (-3)^2} = 4$

$\sec\theta = \frac{-5}{3} = \frac{r}{x}$



$\boxed{\therefore \sin\theta = \frac{4}{5}}$

[3]

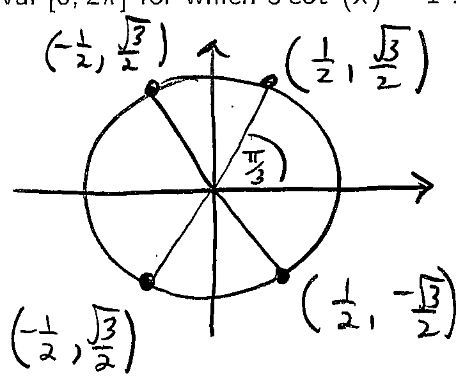
Question 9: Find all values of x in the interval $[0, 2\pi]$ for which $3\cot^2(x) = 1$.

$3\cot^2 x = 1$

$\cot^2 x = \frac{1}{3}$

$\tan^2 x = 3$

$\tan x = \sqrt{3}, -\sqrt{3}$



$\boxed{\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}}$

[4]

Question 10: Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{1}{\sqrt{x+1}}$. Find $(f \circ g)(x)$ and state the domain using interval notation.

$$(f \circ g)(x) = f(g(x))$$

$$= \left. \frac{1}{\left(\frac{1}{\sqrt{x+1}}\right)^2} \right\} *$$

$$= \boxed{x+1}$$

Using $*$, for domain we require $x+1>0$, so $x>-1$, i.e. $\boxed{(-1,\infty)}$.

[5]

Question 11: Find functions f, g and h so that $f(g(h(x))) = \sec^4(\sqrt{x})$.
(There are several possbile correct answers.)

$h(x) = \sqrt{x}$
$g(x) = \sec(x)$
$f(x) = x^4$

or

$h(x) = x$
$g(x) = \sec(\sqrt{x})$
$f(x) = x^4$.

[5]

Question 12: Evaluate the following limit, if it exists: $\lim_{x\to -2} \frac{x^3-2x+8}{x^2-2}$

$$= \frac{(-2)^3 - 2(-2)+8}{(-2)^2-2}$$

$$= \frac{-8+4+8}{2}$$

$$= \boxed{2}$$

[2]

Question 13: Evaluate the following limit, if it exists: $\lim_{t\to 5} \frac{t^2-t-20}{t^2-9t+20}$ $\sim$ "$\frac{0}{0}$"

$$= \lim_{t\to 5} \frac{\cancel{(t-5)}(t+4)}{\cancel{(t-5)}(t-4)}$$

$$= \frac{9}{1}$$

$$= \boxed{9}$$

[4]

Question 14: Evaluate the following limit, if it exists: $\lim_{x\to 4} \frac{1-\sqrt{5-x}}{4-x}$ $\sim$ "$\frac{0}{0}$"

$$= \lim_{x\to 4} \frac{1-\sqrt{5-x}}{4-x} \cdot \frac{1+\sqrt{5-x}}{1+\sqrt{5-x}}$$

$$= \lim_{x\to 4} \frac{1-(5-x)}{(4-x)(1+\sqrt{5-x})}$$

$$= \lim_{x\to 4} \frac{-\cancel{(4-x)}}{\cancel{(4-x)}(1+\sqrt{5-x})} = \boxed{\frac{-1}{2}}$$

[4]