Question 1: Let $f(x) = e^{(2\sqrt{x}+1)}$. Find a formula for $f^{-1}(x)$ (you may assume that the given function $f(x)$ is one-to-one.)

$$y = e^{2\sqrt{x}+1}$$

$$\therefore \ln(y) = 2\sqrt{x}+1$$

$$\ln(y) - 1 = 2\sqrt{x}$$

$$\frac{1}{2}\left[\ln(y)-1\right] = \sqrt{x}$$

$$\left[\frac{\ln(y)-1}{2}\right]^2 = x$$

$$x \leftrightarrow y: \quad \boxed{y = \left[\frac{\ln(x)-1}{2}\right]^2}$$

[5]

Question 2: Use logarithmic differentiation to find y'. Express your answer as a function of x only:

$$y = (\sin x)^{\ln x}$$

$$\ln(y) = (\ln x)\ln(\sin x)$$

$$\frac{1}{y}y' = \left(\frac{1}{x}\right)\ln(\sin x) + (\ln x)\left(\frac{+\cos x}{\sin x}\right)$$

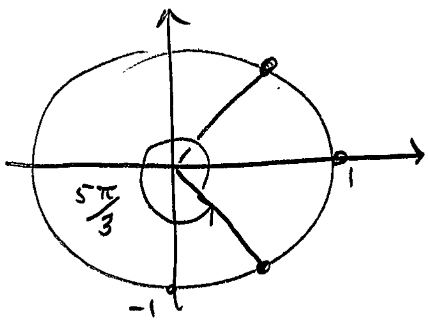
$$\therefore \boxed{y' = (\sin x)^{\ln x}\left[\left(\frac{1}{x}\right)\ln(\sin x) + (\ln x)\left(\frac{\cos x}{\sin x}\right)\right]}$$

$$\underline{\text{or}} \quad \boxed{y' = (\sin x)^{\ln x}\left[\frac{\ln(\sin x)}{x} + (\ln x)(\cot x)\right]}$$

[5]

Question 3:

(a) Determine the exact value of $\cos^{-1}(\cos(5\pi/3))$ $= \cos^{-1}\left(\cos\left(\frac{\pi}{3}\right)\right)$



$= \boxed{\frac{\pi}{3}}$

[3]

(b) Find the derivative of $y = \arctan\left(\sqrt{\sin(\theta)}\right)$.

$$y' = \frac{1}{1+\left(\sqrt{\sin(\theta)}\right)^2} \cdot \frac{1}{2}(\sin\theta)^{-\frac{1}{2}} \cdot \cos\theta$$

$$= \boxed{\frac{1}{1+\sin\theta} \cdot \frac{\cos\theta}{2\sqrt{\sin\theta}}}$$

[3]

(c) Find an equation of the tangent line to $y = \sqrt{1-x^2}\arccos(x)$ at the point where $x = 0$.

At $x=0$, $y=\sqrt{1-0^2}\arccos(0) = 1 \cdot \frac{\pi}{2} = \frac{\pi}{2}$.

$$y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)\arccos(x) + \cancel{\sqrt{1-x^2}}\left(\frac{-1}{\cancel{\sqrt{1-x^2}}}\right).$$

At $x=0$, $y' = \underbrace{\cancel{\frac{1}{2}(1-0^2)^{-\frac{1}{2}}(-2\cdot 0)\arccos(0)}}_{0} + (-1) = -1.$

$\therefore$ Equation of tangent line is

$\boxed{y - \frac{\pi}{2} = (-1)(x-0)}$ or $\boxed{y = \frac{\pi}{2} - x}$

[4]

Question 4: Evaluate the following limits:

(a) $\lim_{x\to 2} \frac{\ln(2x-3)}{x^2-4}$ $\sim$ "$\frac{0}{0}$"

$\stackrel{H}{=} \lim_{x\to 2} \frac{\frac{1\cdot 2}{2x-3}}{2x} = \boxed{\frac{1}{2}}$

[3]

(b) $\lim_{x\to 0} \frac{2-x^2-2\cos(x)}{x^4}$ $\sim$ "$\frac{0}{0}$"

$\stackrel{H}{=} \lim_{x\to 0} \frac{0-2x+2\sin(x)}{4x^3} \sim$ "$\frac{0}{0}$"

$\stackrel{H}{=} \lim_{x\to 0} \frac{-2+2\cos(x)}{12x^2} \sim$ "$\frac{0}{0}$"

$\stackrel{H}{=} \lim \frac{-2\sin(x)}{24x} = \boxed{\frac{-1}{12}}$

[3]

(c) $\lim_{x\to\infty} (e^x+1)^{1/x}$ $\sim$ "∞^0"

$(e^x+1)^{\frac{1}{x}} = e^{(\frac{1}{x})\ln(e^x+1)} = e^{\frac{\ln(e^x+1)}{x}}$.

$\lim_{x\to\infty} \frac{\ln(e^x+1)}{x} \sim$ "$\frac{\infty}{\infty}$"

$\stackrel{H}{=} \lim_{x\to\infty} \frac{\left(\frac{e^x}{e^x+1}\right)}{1} \sim$ "$\frac{\infty}{\infty}$"

$\stackrel{H}{=} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$

$\therefore \lim_{x\to\infty} e^{\frac{\ln(e^x+1)}{x}} = e^1 = \boxed{e}$

[4]

Question 5: For this question use $f(x) = \frac{1}{2}x^2 - 6x + 8\ln(x)$

(a) Determine the intervals on which f is increasing or decreasing.

Here f has domain $(0,\infty)$.

$$f'(x) = x - 6 + \frac{8}{x} = \frac{x^2-6x+8}{x} = \frac{(x-2)(x-4)}{x}.$$

- $f'(x) = 0$? $x=2,\ x=4$
- $f'(x)$ not exist ? no such x.

crit. numbers:	0 (		2		4	
test numbers:		1		3		5
$f'(x) = \frac{(x-2)(x-4)}{x}$:		$+$	0	$-$	0	$+$
$f(x) = \frac{1}{2}x^2 - 6x + 8\ln(x)$:		$\nearrow$	$8\ln(2)-10$	$\searrow$	$8\ln(4)-16$	$\nearrow$

$\therefore$ f is increasing on $(0,2)\cup(4,\infty)$;

f is decreasing on $(2,4)$.

[8]

(b) Determine the local (or relative) maximum and minimum values of f.

f has a loc. max. of $8\ln(2)-10$ at $x=2$;

f has a loc. min. of $8\ln(4)-16$ at $x=4$.

[2]

Question 6: For this question use $f(x) = x - \sin(x)$ on the interval $[0, 3\pi]$

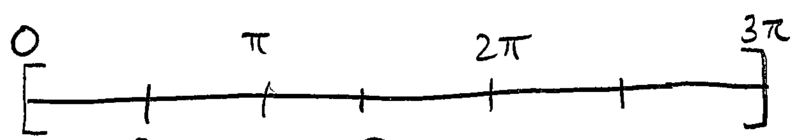
(a) Determine the intervals of concavity.

Domain is $[0, 3\pi]$.

$f'(x) = 1 - \cos(x)$

$f''(x) = \sin(x)$.

- $f''(x) = 0$? $x = 0, \pi, 2\pi, 3\pi$
- $f''(x)$ not exist? no such x.



test. numbers:	$\frac{\pi}{2}$		$\frac{3\pi}{2}$		$\frac{5\pi}{2}$
$f''(x) = \sin(x)$:	+	0	−	0	+
$f(x) = x - \sin(x)$:	$\cup$	π	$\cap$	2π	$\cup$

$\therefore$ f is concave up on $(0, \pi) \cup (2\pi, 3\pi)$;

f is concave down on $(\pi, 2\pi)$.

[8]

(b) Determine all inflection points.

(π, π) , $(2\pi, 2\pi)$.

[2]