Question 1: Let

$$g(x) = \begin{cases} x^4 - cx^2 & \text{if } x < 2 \\ c^2x + 18 & \text{if } x \geq 2 \end{cases}$$

Find the constant c that makes g continuous at all real numbers.

[5]

Question 2: Use the Intermediate Value Theorem to show that the equation $\sqrt{\dfrac{x}{\pi}} = \cos\left(\dfrac{x}{2}\right)$ has a solution on the interval $[0, \pi]$.

[5]

Question 3: Evaluate the following limits, if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning. (Do not use L'Hospital's rule to evaluate limits.)

(a) $\lim_{x\to\infty} \frac{x^2}{\sqrt{x^4+1}}$

[3]

(b) $\lim_{x\to\pi} \frac{\sin x}{(x-\pi)^2}$

[3]

(c) $\lim_{x\to\infty} \sqrt{9x^2+1} - 3x$

[4]

Question 4:

(a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{1}{1+x^2}$. Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

[8]

(b) Now check your work in part (a) by finding $\dfrac{d}{dx}\left[\dfrac{1}{1+x^2}\right]$ using derivative rules.

[2]

Question 5: A ball with an initial velocity of 5 m/s rolls down a hill. The position of the ball after t seconds is $s(t) = 5t + 3t^2$ metres. How long does it take the velocity to reach 35 m/s?

[3]

Question 6: Determine $q''(0)$ if $q(t) = \sec(t)$

[3]

Question 7: Find an equation of the tangent line to $y = \sqrt{1 + 4\sin(x)}$ at the point where $x = 0$.

[4]

Question 8: Find the following derivatives (it is not necessary to simplify your answers):

(a) $y = \dfrac{1 + \sin(x)}{x^2}$

[2]

(b) $f(x) = \left(\sqrt{x} + \dfrac{3}{x}\right)\tan(x)$

[2]

(c) $y = \dfrac{x}{\sqrt{7 - 3x}}$

[3]

(d) $g(t) = \sin(\cos(\tan(t^3)))$

[3]