

# Curve Sketching

So far we have seen that

- (i) If  $f'(x) > 0$  on an interval then the graph of  $y = f(x)$  is increasing on the interval;
- (ii) If  $f'(x) < 0$  on an interval then the graph of  $y = f(x)$  is decreasing on the interval;
- (iii) If  $f''(x) > 0$  on an interval then the graph of  $y = f(x)$  is concave up on the interval;
- (iv) If  $f''(x) < 0$  on an interval then the graph of  $y = f(x)$  is concave down on the interval.

Using this information we then located relative extrema and inflection points, and we sketched a fairly accurate picture of the graph of  $y = f(x)$ .

We now improve our graph by making use of additional information:

- (i) The  $x$ -intercepts of  $y = f(x)$ ,
- (ii) the  $y$ -intercept of  $y = f(x)$ ,
- (iii) the horizontal asymptotes of  $y = f(x)$ , and
- (iv) the vertical asymptotes of  $y = f(x)$ .

## Example 1

Let  $f(x) = \frac{\ln(x)}{x}$ . Sketch the graph of  $y = f(x)$  using the

- (i)  $x$ -intercepts
- (ii)  $y$ -intercepts
- (iii) vertical asymptotes
- (iv) horizontal asymptotes
- (v) intervals of increase/decrease
- (vi) local extreme values
- (vii) intervals of concavity
- (viii) inflection points

**Example 2**

The function  $f(x) = \frac{(x-1)^2}{(x-3)^2}$  has derivatives

$$f'(x) = \frac{-4(x-1)}{(x-3)^3} \quad \text{and} \quad f''(x) = \frac{8x}{(x-3)^4}$$

Sketch the graph of  $y = f(x)$  using the

- (i) x-intercepts
- (ii) y-intercepts
- (iii) vertical asymptotes
- (iv) horizontal asymptotes
- (v) intervals of increase/decrease
- (vi) local extreme values
- (vii) intervals of concavity
- (viii) inflection points