Question 1: Evaluate $\int_{-\infty}^{0} xe^{x}\,dx$ making proper use of required limits.

[5]

Question 2: Use the comparison theorem to determine whether $\int_{1}^{\infty} \frac{\sin^2(x)}{x^2+x^4}\,dx$ converges or diverges.

[5]

Question 3:

(a) Use Simpson's rule on 4 subintervals to approximate $\int_0^1 \sin(\pi t)\, dt$. (Simplify your final answer.)

[5]

(b) Determine an error bound $|E_{S_4}|$ on your approximation in part (a). You may leave your answer in a calculator ready form.

(Recall: the error in using Simpson's rule to approximate $\int_a^b f(x)\, dx$ using n subintervals is at most $\frac{K(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \leq K$ on $[a, b]$.)

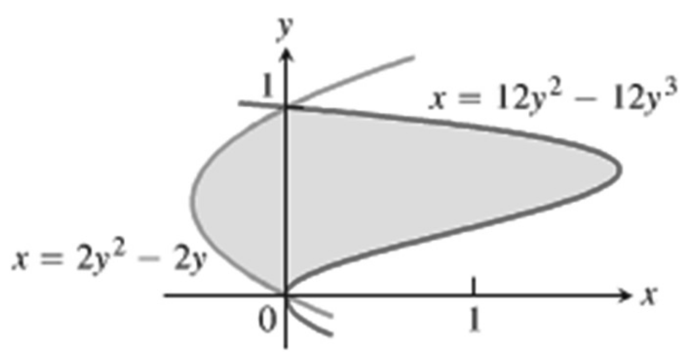
[5]

Question 4:

(a) Determine the area of the region in the first quadrant that is bounded by the curves $y = x^2$, $y = 2 - x$ and $y = 0$.

[5]

(b) Determine the area of the shaded region:



[5]

Question 5:

(a) The region in the first quadrant that is bounded by $y = \ln x$, $y = 0$ and $x = 2$ is rotated about the y-axis. Determine the volume of the resulting solid.

[5]

(b) The flat base of a solid is a circle of radius 1. Parallel cross-sections perpendicular to the base are squares. Determine the volume of the solid.

[5]

Question 6:

(a) The region in the first quadrant that is bounded by the curves $y = e^{x^2}$, $y = 0$, $x = 0$ and $x = 1$ is rotated about the y-axis. Determine the volume of the resulting solid.

[5]

(b) The region in the first quadrant that is bounded by the curves $y = e^{x^2}$, $y = 0$, $x = 0$ and $x = 1$ is rotated about the line $y = 4$. Set up an expression using integrals which represents the volume of the resulting solid. DO NOT EVALUATE YOUR INTEGRALS.

[5]