Question 1: Use the definition of the definite integral in the form

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

to evaluate

$$\int_0^2 (x - x^3)\,dx$$

Carefully set up the Riemann sum and clearly show the steps of your simplification.

[10]

Question 2: Determine the average value of $f(x) = e^{\cos(x)} \sin(x)$ over the interval $[0, \pi/2]$.

[5]

Question 3: A town's population is changing at a rate of $30t(2 - t)$ people per year where $t = 0$ corresponds to the present. Determine the population size at the end of two years if the population is currently 100.

[5]

Question 4: Determine the following integrals:

(a) $$\int \frac{1 + x + \sqrt{x}}{x^2}\, dx$$

[3]

(b) $$\int x^2\sqrt{x^3 + 4}\, dx$$

[4]

(c) $$\int \frac{2x + 5}{x^2 + 5x - 14}\, dx$$

[3]

Question 5: Determine the following integrals:

(a) $\displaystyle\int_{e}^{e^4} \frac{1}{x\sqrt{\ln(x)}}\,dx$

[3]

(b) $\displaystyle\int_{0}^{10} |x-5|\,dx$

[3]

(c) $\displaystyle\int_{0}^{1} \frac{e^x}{1+e^{2x}}\,dx$

[4]

Question 6: Find both the number a and the function f such that

$$6 + \int_a^x \frac{f(t)}{t^2}\, dt = 2\sqrt{x}$$

for every $x > 0$. (Hint: differentiate both sides of the equation.)

[5]

Question 7: Use an area interpretation to determine $\int_0^1 x + \sqrt{1-x^2}\, dx$

[5]