Question 1 [10]: Determine the derivatives of the following functions. It is not necessary to simplify your final answers.

(a)
$$y = \frac{x^3}{3} - 3\sqrt[3]{x} + \sqrt{3}$$

(b)
$$f(x) = (x - \cos x) \left(\frac{4}{x} - \sin x\right)$$

(c)
$$g(x) = \frac{\tan(2x)}{\pi^2 + x^2}$$

[3]

Question 2 [10]: Determine the derivatives of the following functions. It is not necessary to simplify your final answers.

(a) $y = \sec\left(\sqrt{x} - x^5\right)$

(b) $f(\theta) = \sqrt{3\theta - \theta \sin \theta}$

(c)
$$g(t) = \left[t + \cos\left(\frac{1}{\sqrt{t}}\right)\right]^{121}$$

Question 3: Determine the equation of the tangent line to the curve $x^3 - 5xy^2 + y^3 = xy - 13$ at the point (1, 2).

Question 4: There are two tangent lines to the curve y = 1/x which pass through the point (3, -1). Determine the points at which these tangent lines contact the curve.

Question 5: A spotlight on the ground shines on a building wall 12 m away. If a man 2 m tall walks from the spotlight to the building at a speed of 1/2 m/s, how fast is the length of his shadow on the wall decreasing when he is 6 m from the building?



Question 6: Use a linear approximation (or differentials) to approximate $\sqrt[3]{27.1}$.

Question 7: A sphere (ball) of radius r has volume $V = \frac{4}{3}\pi r^3$. Suppose the radius of a sphere is measured to be 10 cm with a maximum measurement error of 1/100 cm. Estimate the maximum measurement error in the calculated volume of the sphere. State units with your answer.