Question 1 [10]: Evaluate the following limits if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning.

(a) $\displaystyle\lim_{x\to 4} \frac{x^2-5x+4}{x^2-3x-4}$

[3]

(b) $\displaystyle\lim_{x\to 16} \frac{4-\sqrt{x}}{x-16}$

[3]

(c) $\displaystyle\lim_{x\to 0} \frac{x}{\sin(x)+\tan(x)}$

[4]

Question 2: Determine $\lim_{x \to 0} x^3 \cos\left(\frac{x+1}{x}\right)$. The Squeeze Theorem may be helpful here. If using the theorem be sure to demonstrate that all requirements of the theorem are satisfied.

[5]

Question 3: Let

$$g(x) = \begin{cases} x^4 - cx^2 & \text{if } x < 2 \\ c^2x + 18 & \text{if } x \geq 2 \end{cases}$$

Find the constant c that makes g continuous at all real numbers.

[5]

Question 4 [10]: Evaluate the following limits if they exist. If a limit does not exist because it is $+\infty$ or $-\infty$, state which with an explanation of your reasoning.

(a) $\lim_{x\to\infty}(\sqrt{x}-x)$

[5]

(b) $\lim_{x\to -6^-}\frac{2x+12}{|x+6|}$

[5]

Question 5 [10]:

(a) Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \dfrac{x}{x+1}$. Neatly show all steps and use proper notation. (No credit will be given if $f'(x)$ is found using derivative rules.)

[8]

(b) Give a one sentence statement describing the relationship between $f'(2)$ and the graph of $y = \dfrac{x}{x+1}$.

[2]

Question 6: Find an equation of the tangent line to $y = 3x^2 - x^3$ at the point where $x = 1$.

[4]

Question 7: Find y' if $y = \dfrac{\sin(x)}{2} - \dfrac{2}{\sqrt{x}}$

[3]

Question 8: Suppose a particle moves along a straight line according to the position function

$$s(t) = t^3 - 12t^2 + 15t + 10$$

When, if ever, is the particle's acceleration zero?

[3]