Question 1: Rationalize the numerator and simplify: $\frac{\sqrt{x}-5}{x-25} \cdot \frac{\sqrt{x}+5}{\sqrt{x}+5}$

$$= \frac{\cancel{(x-25)}}{\cancel{(x-25)}(\sqrt{x}+5)}$$

$$= \boxed{\frac{1}{\sqrt{x}+5}}$$

[3]

Question 2: Simplify: $\frac{\left(1+\frac{3}{c-3}\right) \cdot (c-3)}{\left(1-\frac{3}{c-3}\right) \cdot (c-3)}$

$$= \frac{c\cancel{-3}\cancel{+3}}{c-3-3}$$

$$= \boxed{\frac{c}{c-6}}$$

[3]

Question 3: Simplify: $\sqrt[4]{\frac{t^{1/2}\sqrt{st}}{s^{2/3}}} = \left[\frac{t^{1/2} \cdot s^{1/2} \cdot t^{1/2}}{s^{2/3}}\right]^{\frac{1}{4}}$

$$= \left[t\, s^{-\frac{1}{6}}\right]^{1/4}$$

$$= \boxed{t^{1/4}\, s^{-\frac{1}{24}}}$$

[4]

Question 4: Do the lines $\underbrace{2x - 3y = 4}$ and $\underbrace{x + 3y = 5}$ intersect?

$\therefore y = \frac{2}{3}x - \frac{4}{3}$ $y = \frac{-1}{3}x + \frac{5}{3}$

slopes differ,
so lines must $\boxed{\text{intersect}}$

[3]

Question 5: Find an equation of the line through $(-3, 5)$ that is parallel to the line $x - 2y = 6$

$x - 2y = 6$

$\Rightarrow y = \frac{1}{2}x + 3$

$\therefore$ slope is $m = \frac{1}{2}$ (since lines are parallel)

$\therefore$ Using $m = \frac{1}{2}$ and point $(-3, 5)$:

$$\boxed{y - 5 = \frac{1}{2}(x + 3)}$$

[4]

Question 6: Find an equation of the line through the points $(2, -4)$ and $(-1, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - (-4)}{-1 - 2} = \frac{-5}{3}$$

Using $(2, -4)$: $y - (-4) = \frac{-5}{3}(x - 2)$

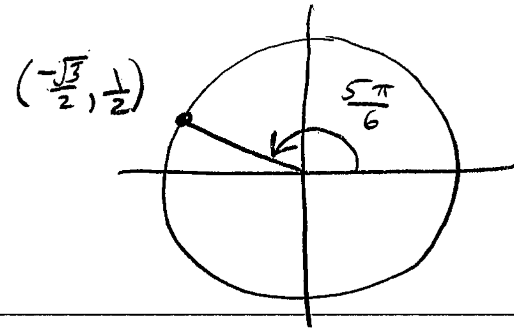
$$\boxed{y + 4 = \frac{-5}{3}(x - 2)}$$

[3]

Question 7: Determine $\sin(5\pi/6) - \tan(5\pi/6)$

$$\sin\left(\frac{5\pi}{6}\right) - \tan\left(\frac{5\pi}{6}\right)$$

$$= \frac{1}{2} - \frac{(1/2)}{-(\sqrt{3}/2)}$$

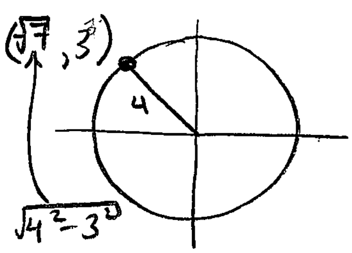


$$= \frac{1}{2} + \frac{1}{\sqrt{3}}$$

$$= \boxed{\frac{\sqrt{3}+2}{2\sqrt{3}}}$$

[3]

Question 8: If $\sin(\theta) = 3/4$ where $\pi/2 < \theta < \pi$ then determine $\tan(\theta)$



$= \sqrt{4^2 - 3^2}$
$= \sqrt{7}$

$\therefore \quad \tan(\theta) = \frac{y}{x} = \boxed{\frac{-3}{\sqrt{7}}}$

[3]

Question 9: Find all values of x in the interval $[0, 2\pi]$ for which $2\cos^2(x) - 1 = 0$.

$$2\cos^2(x) - 1 = 0$$

$$\Rightarrow \cos^2(x) = \frac{1}{2}$$

$$\Rightarrow \cos(x) = \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$\cos(x) = \frac{1}{\sqrt{2}}$ has solutions $\frac{\pi}{4}, \frac{7\pi}{4}$

$\cos(x) = \frac{-1}{\sqrt{2}}$ has solutions $\frac{3\pi}{4}, \frac{5\pi}{4}$

$\therefore \boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

[4]

Question 10: Let $f(x) = x^2 - 3x + 3$. Evaluate and simplify the difference quotient $\dfrac{f(a+h) - f(a)}{h}$.

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{(a+h)^2 - 3(a+h) + 3 - [a^2 - 3a + 3]}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 - \cancel{3a} - 3h + \cancel{3} - \cancel{a^2} + \cancel{3a} - \cancel{3}}{h}$$

$$= \frac{\cancel{h}(2a + h - 3)}{\cancel{h}}$$

$$= \boxed{2a + h - 3}$$

[6]

Question 11: Determine the domain of $g(x) = \dfrac{1}{\sqrt{x}} - \sqrt{3-x}$

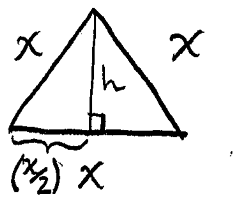
$\dfrac{1}{\sqrt{x}} \Rightarrow x > 0$

$\sqrt{3-x} \Rightarrow 3 - x \geq 0 \Rightarrow x \leq 3$

$\boxed{\therefore \text{ Domain is } (0, 3].}$

[4]

Question 12: Express the area A of an equilateral triangle as a function of its perimeter P.



$$h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \frac{\sqrt{3}}{2}x$$

$$P = 3x \Rightarrow x = \frac{P}{3}$$

$$A = \frac{1}{2}xh = \frac{1}{2}\cdot x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

$$\therefore A = \frac{\sqrt{3}}{4}\left(\frac{P}{3}\right)^2 = \frac{\sqrt{3}\,P^2}{36}$$

$$\therefore \boxed{A = \frac{\sqrt{3}\,P^2}{36}}$$

[5]

Question 13: Let $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 3$. Find $(g \circ f)(x)$ and state the domain.

$$(g \circ f)(x) = g(f(x))$$

$$= \left(\sqrt{x+3}\right)^2 - 3 \quad \Big\} \text{ require } \quad x+3 \geq 0 \Rightarrow x \geq -3$$

$$= \boxed{x, \quad x \geq -3}$$

[3]

Question 14: Let $H(x) = \csc^2\left(\sqrt{x^2+1}\right)$ and $h(x) = x^2$. Find functions f and g so that $H = f \circ g \circ h$. (There are several possible correct answers.)

$$H(x) = \csc^2\left(\sqrt{h(x)+1}\right)$$

$$\therefore \quad g(x) = x+1, \quad f(x) = \csc^2(\sqrt{x})$$

or $\quad g(x) = \sqrt{x+1}, \quad f(x) = \csc^2(x)$

[2]
