Curve Sketching

So far we have seen that

(i) If $f'(x) > 0$ on an interval then the graph of $y = f(x)$ is increasing on the interval;

(ii) If $f'(x) < 0$ on an interval then the graph of $y = f(x)$ is decreasing on the interval;

(iii) If $f''(x) > 0$ on an interval then the graph of $y = f(x)$ is concave up on the interval;

(iv) If $f''(x) < 0$ on an interval then the graph of $y = f(x)$ is concave down on the interval.

Using this information we then located relative extrema and inflection points, and we sketched a fairly accurate picture of the graph of $y = f(x)$.

We now improve our graph by making use of additional information:

(i) The x-intercepts of $y = f(x)$,

(ii) the y-intercept of $y = f(x)$,

(iii) the horizontal asymptotes of $y = f(x)$, and

(iv) the vertical asymptotes of $y = f(x)$.

Example 1

Let $f(x) = \dfrac{\ln(x)}{x}$. Sketch the graph of $y = f(x)$ using the

(i) x-intercepts

(ii) y-intercepts

(iii) vertical asymptotes

(iv) horizontal asymptotes

(v) intervals of increase/decrease

(vi) local extreme values

(vii) intervals of concavity

(viii) inflection points

Example 2

The function $f(x) = \dfrac{(x-1)^2}{(x-3)^2}$ has derivatives

$$f'(x) = \frac{-4(x-1)}{(x-3)^3} \quad \text{and} \quad f''(x) = \frac{8x}{(x-3)^4}$$

Sketch the graph of $y = f(x)$ using the

(i) x-intercepts

(ii) y-intercepts

(iii) vertical asymptotes

(iv) horizontal asymptotes

(v) intervals of increase/decrease

(vi) local extreme values

(vii) intervals of concavity

(viii) inflection points