Question 1: Simplify: $\dfrac{\left(1+\dfrac{2}{c-2}\right)\cdot(c-2)}{\left(1-\dfrac{2}{c-2}\right)\cdot(c-2)}$

$$= \frac{c-2+2}{c-2-2}$$

$$= \boxed{\frac{c}{c-4}}$$

[3]

Question 2: Rationalize the numerator and simplify: $\dfrac{\sqrt{x}-6}{x-36}\cdot\dfrac{\sqrt{x}+6}{\sqrt{x}+6}$

$$= \frac{(x-36)}{(x-36)(\sqrt{x}+6)}$$

$$= \boxed{\frac{1}{\sqrt{x}+6}}$$

[3]

Question 3: Simplify: $\sqrt[4]{\dfrac{t^{1/2}\sqrt{st}}{s^{2/3}}} = \left[\dfrac{t^{1/2}s^{1/2}t^{1/2}}{s^{2/3}}\right]^{1/4}$

$$= \left[t\, s^{-1/6}\right]^{1/4}$$

$$= \boxed{t^{1/4} s^{-\frac{1}{24}}}$$

[4]

Question 4: Find an equation of the line through the points $(2, -4)$ and $(-1, 1)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-4)}{-1-2} = \frac{-5}{3}$$

Using $(2,-4)$: $y-(-4) = \frac{-5}{3}(x-2)$

$$\boxed{y+4 = -\frac{5}{3}(x-2)}$$

[3]

Question 5: Find an equation of the line through $(-3, 5)$ that is parallel to the line $x + 2y = 6$

$x + 2y = 6$

$\Rightarrow y = -\frac{1}{2}x + 3$

$\therefore$ slope is $m = \frac{-1}{2}$ (since lines are parallel)

Using $m = \frac{-1}{2}$ and point $(-3,5)$:

$$\boxed{y-5 = \frac{-1}{2}(x+3)}$$

[4]

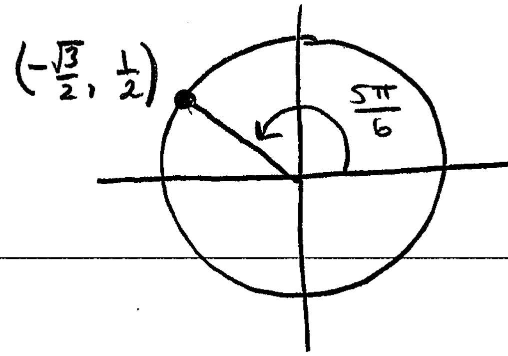
Question 6: Do the lines $2x - 3y = 4$ and $x + 3y = 5$ intersect?

$y = +\frac{2}{3}x - \frac{4}{3}$ $\qquad$ $y = \frac{-1}{3}x + \frac{5}{3}$

slopes differ, so lines are $\boxed{\text{intersecting}}$

[3]

Question 7: Determine $\cos(5\pi/6) - \tan(5\pi/6)$



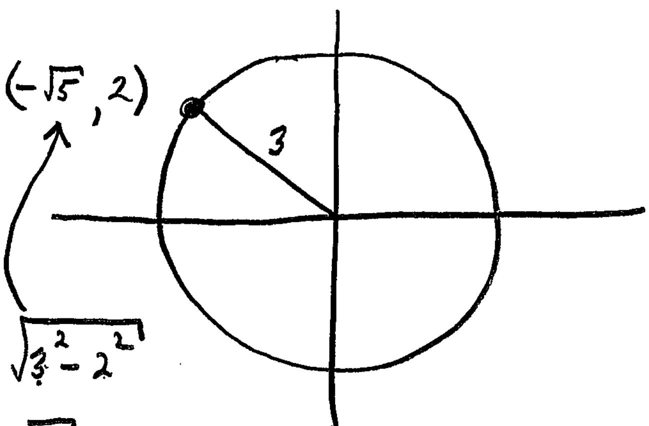
$$\cos(5\pi/6) - \tan(5\pi/6)$$

$$= -\frac{\sqrt{3}}{2} - \frac{(1/2)}{\left(-\frac{\sqrt{3}}{2}\right)}$$

$$= -\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}} = \frac{2-3}{2\sqrt{3}} = \boxed{\frac{-1}{2\sqrt{3}}}$$

[3]

Question 8: If $\sin(\theta) = 2/3$ where $\pi/2 < \theta < \pi$ then determine $\tan(\theta)$



$= \sqrt{3^2 - 2^2}$

$= \sqrt{5}$

$$\therefore \tan(\theta) = \frac{y}{x} = \boxed{\frac{-2}{\sqrt{5}}}$$

[3]

Question 9: Find all values of x in the interval $[0, 2\pi]$ for which $2\cos^2(x) - 1 = 0$.

$$2\cos^2(x) - 1 = 0$$

$$\Rightarrow \cos^2(x) = \frac{1}{2}$$

$$\Rightarrow \cos(x) = +\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}$$

$$\boxed{\therefore \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$$

$\cos(x) = \frac{1}{\sqrt{2}}$ has solutions $\frac{\pi}{4}, \frac{7\pi}{4}$

$\cos(x) = \frac{-1}{\sqrt{2}}$ has solutions $\frac{3\pi}{4}, \frac{5\pi}{4}$

[4]

Question 10: Let $f(x) = x^2 - 2x + 3$. Evaluate and simplify the difference quotient $\dfrac{f(a+h) - f(a)}{h}$.

$$\frac{f(a+h) - f(a)}{h}$$

$$= \frac{(a+h)^2 - 2(a+h) + 3 - [a^2 - 2a + 3]}{h}$$

$$= \frac{\cancel{a^2} + 2ah + h^2 \cancel{- 2a} - 2h \cancel{+3} \cancel{- a^2} \cancel{+ 2a} \cancel{- 3}}{h}$$

$$= \frac{\cancel{h}(2a + h - 2)}{\cancel{h}}$$

$$= \boxed{2a + h - 2}$$

[6]

Question 11: Determine the domain of $g(x) = \dfrac{1}{\sqrt{x}} - \sqrt{4 - x}$

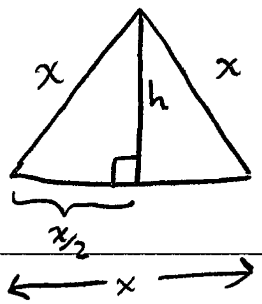
$\dfrac{1}{\sqrt{x}} \Rightarrow x > 0$

$\sqrt{4-x} \Rightarrow 4 - x \geq 0 \Rightarrow x \leq 4$

$\therefore$ Domain is $(0, 4]$.

[4]

Question 12: Express the area A of an equilateral triangle as a function of its perimeter P.



$$P = 3x \Rightarrow x = \frac{P}{3}$$

$$A = \frac{1}{2}xh = \frac{1}{2}\cdot x \cdot \frac{\sqrt{3}}{2}x = \frac{\sqrt{3}}{4}x^2$$

$$\therefore A = \frac{\sqrt{3}}{4}\left(\frac{P}{3}\right)^2 = \frac{\sqrt{3}}{36}P^2$$

$$\therefore h = \sqrt{x^2 - \left(\frac{x}{2}\right)^2} = \frac{\sqrt{3}}{2}x \qquad \boxed{A = \frac{\sqrt{3}}{36}P^2}$$

[5]

Question 13: Let $f(x) = \sqrt{x+3}$ and $g(x) = x^2 - 3$. Find $(g \circ f)(x)$ and state the domain.

$$(g \circ f)(x) = g(f(x))$$

$$= \left(\sqrt{x+3}\right)^2 - 3 \quad \Big\} \text{ requires } \quad x+3 \geq 0 \Rightarrow x \geq -3$$

$$= \boxed{x \quad , \quad x \geq -3}$$

[3]

Question 14: Let $H(x) = \sec^2\left(\sqrt{x^2-1}\right)$ and $h(x) = x^2$. Find functions f and g so that $H = f \circ g \circ h$. (There are several possible correct answers.)

$$H(x) = \sec^2\left(\sqrt{h(x) - 1}\right)$$

$$\therefore g(x) = x - 1, \quad f(x) = \sec^2\left(\sqrt{x}\right)$$

<u>or</u> $\quad g(x) = \sqrt{x-1}, \quad f(x) = \sec^2(x).$

[2]
