

Question 1: Suppose $f(z) = u(x, y) + iv(x, y)$ is entire where $u(x, y) = xy + x + 2y - 5$ and $f(0) = -5 + 2i$. Determine $f(1)$.

[7]

Question 2: Suppose $f(z) = u(x, y) + iv(x, y)$ is analytic in some domain D and that $u(x, y) > 0$ on D . Explain why $\text{Log}|f(z)|$ is harmonic on D . [note: here $|f(z)|$ is real, so $\text{Log}|f(z)| = \ln |f(z)|$.]

[3]

Question 3: Simplify

$$\sin\left(iz - \frac{\pi}{2}\right) + \cos(iz)$$

[5]

Question 4: Determine the largest set on which $f(z) = \text{Log}(4 + i - z)$ is analytic and compute $f'(z)$.

[5]

Question 5: Find all solutions to

$$\text{Log}(z^2 - 1) = \frac{i\pi}{2}$$

[5]

Question 6: Compute both $(i - 1)^{2i}$ and $[(i - 1)^2]^i$ using the principal branch of the logarithm. (Your answers should not be the same. This problem shows that for complex z , α and β , it is not true in general that $(z^\alpha)^\beta = z^{\alpha\beta}$.)

[5]

Question 7: Evaluate $\int_{\Gamma} \operatorname{Im}(z^2) dz$ where Γ is a contour consisting of a line segment from 0 to $1 + i$ followed by a line segment from $1 + i$ to 1.

[6]

Question 8: Evaluate

$$\int_{\Gamma} \frac{e^{1/z}}{z^2} dz$$

where Γ is the arc of the circle $|z| = 4$ from 4 to $-4i$, proceeding counter-clockwise around the origin.

[4]

Question 9: Evaluate

$$\int_C \left[\frac{6}{(z-i)^2} + \frac{2}{z-i} + 1 - 3(z-i)^2 \right] dz$$

where C is a positively oriented circle of radius 1 centre $z = i$.

[5]

Question 10: Evaluate

$$\int_{\Gamma} \frac{3z}{(z+2)(z-1)} dz$$

where Γ is the positively oriented circle $|z| = 4$.

[5]