The final exam will be comprehensive, covering all material since the beginning of the course. In addition to reviewing your homework, I suggest you work through the extra practice problems (from the textbook) that were assigned each week, in particular the problems in Section 6.1 since that section was not covered on our last assignment. Please see below for a list of exam style sample questions. Solutions are attached.

Cheat Sheet and Calculator

A single double-sided letter-size handwritten "cheat sheet" containing formulae, theory and numerical values may be used for the exam. The cheat sheet may not contain worked examples however, and must be submitted when you hand in your test.

A standard non-graphing scientific calculator may be used.

Sample Questions

1. Simplify and express in the form $a+bi$: $\dfrac{5+5i}{(1+3i)(\frac{1}{2}-\frac{i}{2})}$.
2. Determine and sketch all cube roots of $8(1-\sqrt{3}i)$.
3. Evaluate $|e^{iz}|$ if $z = 6e^{i\pi/3}$.
4. Determine the points, if any, at which $f(z) = |\bar{z} - i|^2$ is analytic.
5. Determine the harmonic conjugate of $u(x,y) = 3x^2y - y^3 + x + 4xy$.
6. Can $u(x,y) = xy^2$ be the real part of an entire function? Explain.
7. Find all values of $i^{\mathrm{Log}(i)}$.
8. Evaluate $\displaystyle\int_\Gamma \mathrm{Re}(z)\,dz$ where
   (a) Γ is a line from $z = 0$ to $z = 1+i$
   (b) Γ is a line segment from from $z = 0$ to $z = i$ followed by a line segment from $z = i$ to $z = 1+i$.
9. Evaluate $\displaystyle\int_C \frac{\cos z}{e^z - 1}\,dz$ where C is the circle $|z - 2i| = 1$ traversed once in the positive direction.
10. Evaluate $\displaystyle\int_\Gamma \frac{1}{z}\,dz$ where Γ is any simple contour from $z = -2$ to $z = -i$ which does not leave the third quadrant.
11. Evaluate $\displaystyle\int_C \frac{z^3}{(z+i)(z+2)^2}\,dz$ where C is the circle
    (a) $|z| = 1/2$

(b) $|z| = 3/2$

(c) $|z + 2| = 1/2$

(d) $|z| = 3$

In each case the circle is traversed once in the positive direction.

12. Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in a Laurent series valid for $1 < |z| < 3$.

13. Show that $f(z) = \dfrac{1 + \cos(\pi z)}{(z^2-1)^2}$ has a removable singularity at $z = -1$ (You may use L'Hospital's rule here.)

14. Use the residue theorem to evaluate the following integrals. In each case the circles are traversed once in the positive direction:

(a) $\displaystyle\int_{|z|=2} \frac{z^3 + 2z}{z - i}\, dz$

(b) $\displaystyle\int_{|z|=1} z^2 e^{1/z}\, dz$