1. Determine the set of points on which $f(z) = \left(\frac{(2-i)z+1}{z^2+9i}\right)^3$ is analytic. (Don't use the Cauchy-Riemann equations for this. Rather, use Theorem 3 in 2.3).

2. For each of the following determine the set of points on which the function is differentiable:

   (a) $f(z) = e^{x^2-y^2}\cos(2xy) + ie^{x^2-y^2}\sin(2xy)$

   (b) $f(z) = \frac{x^3+xy^2+x}{x^2+y^2} + i\frac{x^2y+y^3-y}{x^2+y^2}$

3. Find real constants a, b, c and d so that $f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ is entire.

4. The function $f(z) = x^2 - x + y + i(y^2 - 5y - x)$ is not analytic at any point, but is differentiable along a line in the complex plane. Find the line.

5. Show that $u(x, y) = \ln(x^2 + y^2)$ is harmonic in an appropriate domain D and then find a harmonic conjugate $v(x, y)$ for $u(x, y)$.

6. (a) Show that $v(x, y) = \frac{x}{x^2+y^2}$ is harmonic on $D = \mathbb{C} \setminus \{0\}$.

   (b) Find a function $f(z) = u(x, y) + iv(x, y)$ that is analytic on D .

   (c) Express $f(z)$ in (b) in terms of the variable z.